GROWTH AND YIELD OF DOUGLAS FIR PLANTATIONS IN THE CENTRAL NORTH ISLAND OF NEW ZEALAND

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with 2 diskettes
in back pocket

To my parents

whose efforts to my earlier education

made this thesis possible

Abstract

Three growth and yield models have been developed for Douglas fir plantations in the Central North Island of New Zealand: DFCNIGM1, DFCNIGM2 and DFCNIGM3.

DFCNIGM1_____ Douglas Fir Central North Island Growth Model version 1, is a whole stand model for Central North Island plantations, that includes Kaingaroa, Pureora, Waimihia and Whirinaki. Data for Karioi and Whakarewarewa forests were also available but were excluded from the model, because their growth trends differed from the rest, they are separated geographically from the main block of forests and their plot data base was not big enough to allow adequate validation of their inclusion. The version 1 model, which was completed and put into routine operation in March 1989, consists of three parts: a growth and yield projector for (I) healthy, (II) diseased and thinned, and (III) diseased and unthinned stands. Each part consists of a basal area projection equation (Schumacher), a stand volume (combined variable) equation, and a merchantable volume equation (being a function of total volume). All three share the same site index equation (Chapman-Richards) and mortality equation (a simple decay function).

Inputs to the model are initial age, initial stems/ha, initial basal area per hectare, site index (or mean top height) and values specified for proposed thinning operations. Outputs of future values generated by the model are basal area/ha, volume/ha, merchantable volume/ha to a

15 cm top diameter limit, stems/ha, mean top height and quadratic mean diameter of the stand.

The version 1 model has been in use by New Zealand Forestry Corporation, Timberlands Ltd, since its completion and proved able to give good prediction for all stand statistics except regular mortality, which statistic appears to have been over-estimated. For this reason, the model was subsequently revised into DFCNIGM2____ Douglas Fir Central North Island Growth Model version 2, in October 1989, with the original mortality equation replaced by a new one. The new mortality equation was derived by establishing a rate of change in stems/ha over time as a function of stems/ha at beginning of the growth period, basal area/ha, dbh and site quality; mortality was obtained by integrating this rate function. This equation reduced the residual sums of squares by 37% in comparison with the original one.

Although the basal area projection equations in DFCNIGM1 predict the future basal area/ha well, they were replaced with an even better-fitting Hossfeld equation. The Hossfeld equation has a more desirable property than the Schumacher: at age zero its yield is equal to zero, whereas it is not defined in Schumacher's equation.

DFCNIGM3_____ Douglas Fir Central North Island Growth Model version 3, is a diameter distribution model for the same plantations resources. This model consists of two parts, namely separate growth and yield projectors for healthy and diseased stands. Each part consists of a stand level and a diameter distribution model. The stand level model is basically the same as DFCNIGM2, and the diameter distribution is generated from stand variables.

The reverse Weibull distribution was used as the probability density function for characterising the distribution of diameters at breast height over bark (dbhob) using maximum dbhob, arithmetic mean dbhob and standard deviation of dbhob as state variables. The maximum diameter and standard deviation projection equations were based on the Hossfeld function. Estimating the moments using difference equations makes use of initial values which are generally available in permanent sample plot (PSP) data and which can improve the fit substantially. The b and c parameters of the reverse Weibull distribution were estimated in the usual manner. The location parameter "a" was estimated through the type III Extreme Value Distribution and an extreme percentile calculated from the return period.

Based on the stability postulate by Fisher and Tippett (1928) and comments from Gumbel (1958), it was shown that the type III Extreme Value Distribution should be used as the extreme value distribution when a reverse Weibull function is used as the initial distribution.

Maximum diameters of each of the PSP plot measurements were extracted. Those maximum diameters were sorted by age, then the arithmetic mean, standard deviation and the maximum of the maximum diameters of each age class were calculated. Moments of the type III Extreme Value

Distribution, i.e. the arithmetic mean, standard deviation of the extreme largest diameters, for generating the parameters of the type III Extreme Value Distribution were fitted to those calculated values just described using the Hossfeld function. The scale parameter b_x and shape parameter c_x of the type III Extreme Value Distribution are calculated using method of moments. The location of the type III Extreme Value Distribution is set equal to the projected maximum of the largest diameters. With parameters a_x, b_x and c_x of the type III Extreme Value Distribution determined, the location of the reverse Weibull distribution was obtained by

$$a = a_x - b_x[-\log(p)]^{1/cx}$$

Where the percentile p is calculated from the return period. Theoretically, p should be the 100th percentile on the extreme value distribution, which would ensure that the maximum diameter (or the location parameter of the initial distribution) chosen will not be exceeded at a specified age. In practice, the 100th percentile might not be able to produce a good fit and some lower percentile can be tried. But how much lower? Based on the return period, it is shown that it can be chosen between the 95 and 100 percentiles, and still ensure that the maximum diameter chosen will not be exceeded for a specified age, while producing a better fit. Thus by combining the extreme percentile with the extreme value distribution, a good fit can be obtained and bias can be safely avoided; this cannot be achieved if the percentile on the extreme value distribution is chosen arbitrarily.

According to the symmetry principle, the Weibull itself should be used as the extreme value distribution when one works with the smallest diameter. Thus the proposed procedure can be applied to the conventional Weibull approach by using the Weibull distribution as the extreme value distribution, which is also called a type III Extreme Value Distribution (Gumbel, 1958).

A modified linear equation was used to model the height corresponding to mid-point diameters. Existing volume and taper functions currently used to generate PSP results were employed for the purpose of this study. These functions could be the subject of further investigation to see if useful refinements to them could be made. In version 3 of DFCNIGM, thinned and unthinned data are pooled and the thinning effect on yield is treated by introducing a thinning index to relevant equations and fitting them to the pooled data.

Inputs and outputs of DFCNIGM1 and DFCNIGM2 are the same as for most New Zealand growth and yield models and have been described elsewhere (Liu Xu, 1989). Inputs to DFCNIGM3 are age, site index or mean top height, initial basal area/ha, initial stems/ha, maximum dbhob, standard deviation of dbhob, initial extreme largest diameter, initial mean extreme largest diameter, standard deviation of the extreme largest diameter and values for specified thinning operations. Outputs are dbh class, number of trees/ha, mean height, volume and merchantable volume per hectare of each class plus basal area/ha calculated from a stand basal area equation that is compatible with that summed up from all the dbh classes.

Preliminary verification and validation indicated that the proposed models give good predictions of both stand and diameter distributions statistics.

Contents

CHAPTER 1 INTRODUCTION

1.1 BACKGROUND

Douglas Fir (Pseudotsuga menziesii (Mirbel) Franco) has been grown in New Zealand for about 100 years now. According to Kirkland (1969) the species was introduced as a plantation crop in New Zealand about 1897, when trials of several exotic tree species, including Douglas fir, were carried out in Kaingaroa. But its widespread establishment rather than just in small woodlots started about 1920. By 1929 an average of 810 ha had been planted annually and 6650 hectares of plantations had been formed in northern Kaingaroa alone (Kirkland, 1969; Wilcox 1978)

Until 1927, no exact records of seed origin were kept. However, accurate seed sources were subsequently recorded for all imported Douglas fir seedlots. It is commonly believed (Wilcox, 1978; James and Bunn, 1978) that plantings between 1915 and 1928 originated from Washington State. Importations since have been mostly from Washington State, Oregon, British Columbia and California.

Douglas fir is considered to have good timber qualities, particularly for engineering purposes (Hellawell, 1978), or at least as good as those of radiata pine (James and Bunn, 1978). Unfortunately the species became infected in 1960's by Phaeocryptopus gaeumannii (Hood and Kershaw, 1973, 1975), an ascomycetous fungus that parasitises needles of Douglas fir, and subsequently reduces its growth rate (Beekhuis, 1978; James and Bunn, 1978).

P. gaeumannii was first noticed in New Zealand at Taupo in 1959. A nation-wide survey indicated that the infected area was confined within a 130 km radius and no symptoms of ill health were reported at the time. By 1974 the fungus had spread throughout North Island Douglas fir plantations. It was found in the South Island in 1969 near Nelson and by 1974 it had spread to Marlborough, Westland and Canterbury (Hood et al., 1975; Gilmour, 1966). Chlorosis and needle loss in Central North Island were first reported in 1962. By 1973 volume increment loss in Kaingaroa region was estimated to be 140 000 m^3 annually (James and Bunn, 1978).

The propensity to disease infection has been influenced by the silviculture practised on this species. Douglas fir branch growth follows a radial pattern producing branches which, if allowed to grow freely, will be large; consequently log degrade will occur. To restrict the branch index (mean diameter of the largest branches per log) on the first log to 2.5 cm, a high initial stocking of 1700 to 2000 stems/ha is required, and tight control of stand density maintained thereafter. It has been suggested that P. gaeumannii was not solely responsible for the growth decline as insects, drought and silvicultural neglect were also involved. All these factors together, particularly high stand densities, put stands in a state of stress which made the trees more susceptible to pathogens. Regular heavier thinnings have subsequently been introduced to maintain deep effective crowns. Most stands are now thinned two or three times between age 25 and 45 to between 170 and 600 stems/ha (James and Bunn, 1978; Kirkland, 1969).

Table (1.1) Sample Plot Statistics (n=2319)

Variables	**Maximum**	**Minimum**	**Mean**
age	82	9	35
net basal area (m^2/ha)	158.2	3.3	44.9
net volume (m^3/ha)	1949.1	11.5	481.1
merch. volume (m^3/ha)	1861.6	0.6	425.1
dbh (cm)	66.8	5.8	32.5
h_{100} (m)	46.3	9.0	25.0
N/ha	4941	44	638
initial N/ha	6944	1376	2509
altitude (m)	930	0	473
number of thinnings	4	0	2
site index (m)	41.2	19.5	31.1

An indication of yields and other average measures of crop production for Douglas fir in the Central North Island, summarised from the data set used in this study, is given in Table 1.1. As the table shows, plots were located in stands which were established at a very high initial stocking level of up to 6944 stems per unit area with an average of 2500 stems/ha. The stands were then left unthinned or thinned up to 4 times to an average stems per hectare varying from 170 to 600 (one plot having 44 stems/ha has been ignored). On average, when stands reach an age of 35 years, diameter at breast height will be about 30 cm and mean top height about 25 m with a basal area of 45 m^2/ha and volume of 480 m^3/ha. The yield per unit area may be lower now than the table shows since this data set includes some pre-disease measurements.

Although much less important than radiata pine, Douglas fir is the second ranking exotic species in New Zealand. Currently, Douglas fir plantations occupy 63 130 hectares of New Zealand's forest estate, of which there are more than 24 700 ha, about 40 %, in the Central North Island region (Turland and Novis 1990), making up the second largest portion of exotic forests in this country. Yet much less research has been done on this species compared with Pinus radiata, partly because of its decrease in growth as a result of disease and its consequent comparative loss of productivity relative to radiata pine.

However, as large amounts of the resource exist and new plantings are still being added each year, it would be unwise not to recognise the importance of managing this species. Sound forest management decision-making has always depended on accurate growth and yield forecasts. Thus, development of computerised growth and yield models for Douglas fir plantations is essential for future management of this substantial part of New Zealand's plantation resource.

Before this present study was carried out, the only other growth and yield model for North Island Douglas fir plantations was that developed by Mountfort (1978) for Kaingaroa forests. His summary of the steps and functions used in the model is as follows,

(1) select starting points of initial stems per hectare, height, basal area per acre, and site index;

(2) stems lost during the ensuing year are calculated from the mortality relationships in one of three ways

(a) if the relative spacing (see Beekhuis, 1966) is greater than or equal to 18%, there is no mortality,

(b) if the relative spacing is less than or equal to 9%, then mortality is linear as defined by the 9% relative spacing line,

(c) if the relative spacing lies between 9% and 18% relative spacing, then the mortality loss follows the arc of a circle which is tangential to the 9% relative spacing line and the line of no mortality corresponding to the particular stocking;

(3) stems lost are then converted to basal area lost;

(4) annual gross basal area/acre increment (G/T) is calculated from the following relationships

(a) up to stand height 39 feet (11.9 metres):

$$\ln(G) = 2.136\ln(\text{Initial } h_{100})-3.249 \quad \text{...........(1.1.1)}$$

gross basal area increment

$$\Delta G = G_2-G_1 \quad \text{..................................(1.1.2)}$$

(b) Stand heights 40 feet and over:

ΔG = S.I.F.(921.362(1/Initial h_{100})-1.303)

..............................(1.1.3)

Where

S.I.F = 0.01(1.528(S.I.)-62.380);

G = basal area/acre;

h_{100} = initial mean top height;

$\Delta G = G_2 - G_1$;

S.I.F. = site index factor;

S.I. = site index.

(5) basal area/acre lost is subtracted from gross basal area to give net basal area;

(6) net basal area/acre is added to initial value to give net basal are/acre at the end of the growth period;

(7) total stem volume/acre is calculated from a combined variable stand volume function

$V = G(0.337h_{100}+1.789)$ ....(1.1.4)

(8) recoverable volume V_m is derived from total stem volume by the relationship:

$V_m = V(0.038d_g + 0.166)$ ....(1.1.5)

when mean tree d.b.h., d_g, is less than 18 inches (45.7 cm); or is fixed at 0.85 × total stem volume when mean tree d.b.h. is 18 inches or more;

(9) the process is repeated annually for the required number of years;

(10) to predict the residual G, N, and V after thinning specified in terms of basal area/acre removed, the desired reduction is translated into stems/acre using the following

relationships

(a) First thinnings:

$$\% N\ cut = 7.2348 + 1.3744(\%G\ cut) - 0.0047(\%G\ cut)^2 \quad \ldots\ldots\ldots\ldots(1.1.6)$$

(b) Second and subsequent thinnings:

$$\% N\ cut = 1.4917 + 1.3703\ (\%G\ cut) - 0.0035\ (\%G\ cut)^2 \quad \ldots\ldots\ldots\ldots(1.1.7)$$

where thinning yields in terms of volume are calculated from the stand volume regression in the same ways as in (1.1.4) above.

(11) Gross ΔG for the 3 year period after thinning is adjusted by

$$\%\Delta G = 100.1030 + 0.5036(\%G\ cut) - 0.0170\ (\%G\ cut)^2 \quad \ldots\ldots\ldots(1.1.8)$$

There are major theoretical and practical deficiencies in this previous model, as explained below.

(1) The yield equations used by Mountfort are illogical and inconsistent. An appropriate function, such as the one (1.1.9) used by Clutter (1963), Sullivan and Clutter (1972); Clutter <u>et al</u>., (1983).

$$\ln(G_2) = \ln(G_1)\left(\frac{T_1}{T_2}\right) + \alpha\left(1-\left(\frac{T_1}{T_2}\right)\right) + \beta S\left(1-\left(\frac{T_1}{T_2}\right)\right) \quad \ldots\ldots\ldots(1.1.9)$$

should have following properties:

(a) as T_2 approaches T_1, $\ln(G_2)$ approaches $\ln(G_1)$ and any projection model lacking this property is illogical (Clutter <u>et al</u>., 1983);

(b) as T_2 approaches ∞, $\ln(G_2)$ approaches an asymptote,

$\alpha + \beta S$. i.e. its growth will reach an upper limit at a certain age;

(c) projections are path invariant. [i.e. projection made from T_1 to T_3 is equal to that made from T_1 to T_2 then from T_2 to T_3, and projection models that lack this property are inconsistent (Clutter et al., (1983)];

(d) the shape of the cumulative growth is sigmoid.

Yield equations in Mountfort's Douglas fir model possess none of those essential properties.

(2) Gross basal area does not form a smooth curve.

Gross basal area should following a smooth single sigmoid curve for a stand (Clutter et al., 1983; Avery and Burkhart, 1983). But in step (4) of the construction of Mountfort's model, two different functions were used to calculate the gross basal area increment. If the stand height is under 40 feet, use (1.1.1); and if the stand height is greater than or equal to 40 feet, then use (1.1.3). The curves of basal area increment drawn using the values calculated from these two equations would not necessarily join at the point of stand height 40 feet. This means that the growth of the stand is not represented by a smooth single curve as it theoretically should be.

(3) Lack of desirable properties in mortality projection equation.

Any mortality equation should possess the following properties (Clutter et al., 1983):

(a) if T_2 equals T_1, N_2 should equal N_1;

(b) for even-aged stands, if T_2 is greater than T_1, N_2

should be equal to or less than N_1;

(c) for even-aged stands, as T_2 becomes very large, N_2 should approach a lower asymptote;

(d) projection made from T_1 to T_3 is equal to that made from T_1 to T_2 then from T_2 to T_3.

Mountfort's mortality function has none of these properties. Mortality follows a straight line when relative spacing is less than 9% and follows an arc of a circle tangential to the 9% relative spacing line and the line of no mortality corresponding to a particular stocking. Thus, as in the yield equations, mortality is represented by more than one curve and does not follow a single smooth line.

(4) Mortality prediction errors in the model are also compounded in the yield equation. If a volume equation independent of stocking is used in a model, it is possible to produce a precise yield equation regardless of how mortality is calculated. Otherwise any error resulting from mortality prediction would be compounded within the yield equation. In Mountfort's Douglas fir model, volume was calculated by an equation that is implicitly dependent on stocking, which is a variable difficult to model and in which prediction errors are mostly large. The stems/ha lost during the ensuing year were first calculated, then converted to basal area lost. Basal area lost was subtracted from gross basal area to get net basal area; net basal area is added to initial basal area to give basal area/acre at end of the growth period. This basal area/acre is in turn used to calculate total stem volume/acre. As can

be seen, the mortality prediction errors have been carried all the way through to the final yield equation.

Also, DFIR was developed for pre-diseased stands. It would not be much use for today's management of the plantations as all Central North Island Douglas fir plantations have now been infected by the disease. Many additional data have been collected since the completion of this previous model 12 years ago and an update is necessary. This study, therefore, is aimed at producing a biologically and mathematically sound model of growth and yield of Central North Island Douglas fir crops, one that is sensitive to managerial requirements for modern yield forecasting.

1.2 THE SCOPE OF THE STUDY

Permanent Sample Plot (PSP) data available for this study of growth and yield of Douglas fir come from seven forests in Central North Island: Kaingaroa, Whakarewarewa, Waimihia, Whirinaki, Horohoro, Karioi and Pureora. The broad aim of this research was to develop both whole stand and diameter distribution models. No attempt has been made to develop an individual tree model because there is no apparent managerial need for it and suitable data were not available either. The data set is such that the various models can be safely applied only between ages 15 and 70 years, and only for Kaingaroa and surrounding forests.

1.3 OBJECTIVES

The detailed objectives of this study are to

(1) develop whole stand and diameter distribution growth and yield models for Douglas fir plantations in the Central North Island of New Zealand;

(2) identify the presence or absence of intra-regional or temporal differences among subsets of the data and make due allowance for such effects;

(3) check on the evidence of bias, test for the reliability and applicability of component equations and validate the performance of the model within a specified range of operating conditions;

(4) prepare computer programs to allow the models to be run on IBM compatible micro-computers.

CHAPTER 2 REVIEW OF PERTINENT LITERATURES

2.1 REVIEW OF INTERNATIONAL GROWTH AND YIELD MODELLING METHODOLOGY

2.1.1 Methodology

2.1.1.1 Normal yield tables

Earliest yield predictions date back at least to the beginning of the nineteenth century, when German foresters used the normal yield table approach to predict yield of even-aged stands.(Bickford et al., 1957; Fries, 1967; Curtis, 1972; Avery and Burkhart, 1983; Clutter et al., 1983; Burkhart et al., 1987). The application of normality concept in European forest management was a result of fear of inadequate wood supply (Whyte, 1990). This approach, however, has several limitations (Ware et al. 1988).

Firstly, the graphical procedure used in the earliest tables constrains the number of variables used in the analysis, and so analysts had to hold density at a fully stocked level. With modern computer technology and analytical techniques, such constraints no longer exist.

Secondly, since this approach assumes that stands are fully stocked, adjustment is needed when the table is used to predict yield if they are not.

Thirdly, it is commonly believed that normal yield tables do not represent a rational management goal, because a non-fully stocked stand with proper silviculture treatments could well produce higher returns than a fully-stocked untended stand could (Curtis, 1972).

Finally, the definition of normality is subjective. Even though, for some timber types, it might be the only method available (Avery and Burkhart, 1983), this approach has been made a historical artifact rather than a dynamic theory because of the subjective nature of its definition and the availability of some improved methods. (Nelson and Bennett, 1965; Curtis, 1972; Avery and Burkhart, 1983; Clutter, et al., 1983).

A similar approach is the so called empirical yield table (Schumacher, 1939; Bennet et al., 1959; Chambers and Wilson, 1972; Avery and Burkhart, 1983). Its advantage over normal yield tables is that it applies to average rather than fully stocked stands, which thus eliminates the problem of defining normality. Adjustment still has to be made when the tables are applied to stands that are not 'average'. Because of this, it too is seldom used today.

2.1.1.2 Variable density formulations

A major development was the variable density growth and yield formulation first suggested by MacKinney et al., (1937) and applied by MacKinney and Chaiken, Schumacher. (1939). For example, an equation used by MacKinney et al., was

$$\log(Y) = \beta_0 + \beta_1 T^{-1} + \beta_2 S + \beta_3 \log(SDI) + \beta_4 C \quad \ldots(2.1.1)$$

where Y is yield, T is stand age, S is site index, SDI is Reineke's (1933) stand density index and C is composition index (basal area per unit area divided by total stand basal area).

In this approach, stand density is a dynamic part of the prediction system and multiple regression analysis is used to fit the equations and estimate the parameters.

Apart from expressing density as an independent variable, other properties of the equation include logarithmically expressing yield as the dependent variable and the differential effect of independent variables on yield; use of the reciprocal of age expresses an asymptotic effect of age on yield. (Langdon, 1961; Vimmerstedt, 1962; Avery and Burkhart, 1983; Clutter et al., 1983; and others).

Since then, many investigators have used this method to construct growth and yield equations (e.g. Wenger et al., 1958; Bennett et al., 1959; Brender, 1960; Schumacher and Coile, 1960; Langdon, 1961; Nelson, et al., 1961). However, the method these later workers used was still akin to that of MacKinney, Schumacher and Chaiken (1937), MacKinney and Chaiken (1939) and Schumacher (1939).

2.1.1.3 Compatible growth and yield models

Buckman (1962) and Clutter (1963) were the first researchers to express the biological relationship between growth and yield by logical mathematical relations between differentiation and integration. Yield was obtained by integrating a growth function to form a mathematically compatible growth and yield model. Sullivan and Clutter (1972) further refined the concept and produced a simultaneous yield model by simultaneously estimating yield and cumulative growth as a function of initial age, basal area, site index and future age. When future age equals current age, the equation is reduced to a conventional yield model. Thus, it is simultaneously a yield equation for current condition and a projection model for the future. The yield equation (using conventional IUFRO symbols) was

$$\ln(V) = \beta_0 + \beta_1 S + \beta_2 T^{-1} + \beta_3 \ln(G) \quad \ldots\ldots\ldots\ldots\ldots(2.1.2)$$

Differentiation of (2.1.2) with respect to age presents relative rate of volume growth:

$$\frac{dV/dT}{V} = -\frac{\beta_2}{T^2} + \beta_3\left(\frac{dG/dT}{G}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1.3)$$

They both used the Schumacher equation to project basal area:

Differentiation of this equation results in

$$\ln(G)=\alpha_0+\alpha_1 S+\frac{\alpha_2}{T}+\frac{\alpha_3\ln(G_{20})}{T}+\frac{\alpha_4 S}{T}\ldots\ldots\ldots\ldots\ldots\ldots(2.1.4)$$

$$\frac{dG}{dT}=\frac{G}{T}[\alpha_0+\alpha_1 S-\ln(G)]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1.5)$$

Integration of (2.1.5) gives the basal area projection equation

$$\ln(G_2)=\ln(G_1)\left(\frac{T_1}{T_2}\right)+\alpha_0\left(1-\left(\frac{T_1}{T_2}\right)\right)+\alpha_1 S\left(1-\left(\frac{T_1}{T_2}\right)\right)\ldots\ldots(2.1.6)$$

This equation represents a sigmoid curve, has an inflection point and an asymptote. It is mathematically compatible, numerically consistent and path invariant.

To predict future volume, equation (2.1.2) can be written as

$$\ln(V)=\beta_0+\beta_1 S+\frac{\beta_2}{T}+\beta_3\ln(G)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1.7)$$

Substituting (2.1.6) for G gives the volume projection equation

$$\ln(V_2)=\beta_0+\beta_1 S+\frac{\beta_2}{T}+\beta_3\ln(G_1)\left(\frac{T_1}{T_2}\right)+\beta_4\left(1-\left(\frac{T_1}{T_2}\right)\right)+\beta_5 S\left(1-\left(\frac{T_1}{T_2}\right)\right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1.8)$$

where

$\beta_4 = \beta_3\alpha_0$

$\beta_5 = \beta_3\alpha_1$

2.1.1.4 Diameter distribution modelling

Another major development in the history of yield prediction is diameter distribution modelling.

Foresters use of mathematical functions to project diameter distributions originated as early as 1898, when de Liocourt constructed a diameter distribution model for all-aged stands using geometric progression (Meyer and Stevenson, 1943). In 1930, Meyer published his studies on diameter distribution for even-aged stands. He applied this approach to loblolly pine in 1942. Based on de Liocourt's idea, Meyer and Stevenson (1943) suggested the exponential function for expressing diameter distribution of uneven-aged stands. It was found satisfactory when this model was later used by Meyer (1952), Schmelz and Lindsey (1965). Since that time, a great deal of attention has been paid to diameter distribution modelling because this type of model can provide more detailed information about stand structure that is often critical to management decision-making. Examples of stand models that generate diameter distribution are Bennett and Clutter (1968), Lenhart and Clutter (1971), Lenhart (1972), Burkhart and Strub (1974), Smalley and Bailey (1974), Knoebel, et al., (1986), Baldwin and Feduccia (1987), Matney, et al., (1987), Lenhart (1988) and Bowling et al., (1989). For intensively managed plantations, the use of diameter distribution models is to be preferred to stand level models.

Numerous probability density functions have been used to formulate the diameter distribution. For example, Gram-Charlier series (Meyer 1930), Pearl-Reed growth curve (Osborne and Schumacher, 1935; Nelson, 1964), Pearsonian curves (Schnur, 1934), Johnson's S_b distribution (Hafley and Schreuder, 1977), three-parameter logarithmic-normal (Bliss and Reinker 1964), Beta distribution (Clutter and Bennett, 1965; McGee and Della-Bianca, 1967; Lenhart and Clutter, 1971), Gamma distribution (Nelson 1964) and the Weibull distribution (Bailey, 1972; Bailey and Dell, 1973; Borders et al., 1987). The Weibull function is the most extensively used function in recent diameter distribution modelling since its introduction to growth and yield modelling by Bailey in 1972 (Bailey and Dell, 1973; Schreuder et al., 1979; Matney and Sullivan, 1982; Burk and Burkhart, 1984).

The popularity of the Weibull function stems from its having the following desirable properties:

(1) it is suitable for the cases where the conditions of "strict randomness" of the exponential distribution are not satisfied (Bailey, 1972; Bailey and Dell, 1973; Johnson and Kotz, 1970; Knoebel and others 1986), such as systematically correlated PSP sample;

(2) the flexibility of the model (Bailey and Dell, 1973; Johnson and Kotz, 1970);

(3) it has a closed form (Bailey and Dell, 1973; Clutter et al., 1978);

(4) relative ease of mathematical manipulation (Bailey and Dell, 1973).

In early work, parameters of the various density functions were estimated directly as functions of stand attributes (for example, age, stems/ha, site index), using regression techniques. Diameter distribution models so constructed, did not necessarily produce the same yield estimates as that given by the whole stand model. For example, Clutter and Belcher (1978) developed a model for slash pine plantation, in which the parameters of the Weibull were predicted using the following equations,

$$a = 8.04979 - 0.1015176T + 0.131034h_{100} - 3.04792\ln(h_{100}) \quad \text{.........................(2.1.9)}$$

$$b = -3.84157 + 0.05628T + 481.30737/N + 1.9111\ln(h_{100}) \quad \text{.........................(2.1.10)}$$

$$c = 3.6261 + 9.4599/T \quad \text{.........................(2.1.11)}$$

Correlation coefficients, R^2, of these equations are 0.107, 0.357 and 0.020 respectively. Low R^2 values were primary evidence of major lack of fit. This led to the development of the parameter recovery procedures in which parameters of the probability function are estimated implicitly from stand estimates. Models developed in this manner are mathematically compatible between whole stand and diameter distribution models, and numerically consistent among various stand yield estimates. (Hyink, 1980; Frazier, 1981; Matney and Sullivan, 1982; Cao et al., 1982; Hyink and Moser, 1983; Cao and Burkhart, 1984; Knoebel et al., 1986; Borders et al., 1987; Bowling et al., 1989). The other advantage of using parameter recovery methods is that stand measures such as basal area and diameter percentiles can be estimated with much higher precision than can the pdf parameters (Borders et al., 1987).

An example of such a model would be that by Matney et al., (1987), in which arithmetic mean diameter (d_a), quadratic mean diameter (d_q) and minimum diameter (d_{min}) were predicted by equations

$$d_a = \exp[4.121 - 0.2801\ln(N) - 31.51/h_{100}] \ldots\ldots (2.1.12)$$

$$d_q = d_a[1 + (0.0955 + 0.1802(h_{100}/d_a))^2]^{0.5} \ldots\ldots\ldots (2.1.13)$$

$$d_{min} = \exp(-0.324 + 0.2006d_a) \ldots\ldots\ldots\ldots\ldots\ldots (2.1.14)$$

with R^2 of 0.8757, 0.9954 and 0.2880 respectively. The following equations were then solved for the parameters a, b and c

$$a = d_a - \frac{(d_a - d_{min})}{[1 - \frac{1}{n^{(1+\frac{1}{c})}}]} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1.15)$$

$$b = (d_a - \frac{d_{min}}{\Gamma(1+\frac{1}{c})})[1 - \frac{1}{n^{(1+\frac{1}{c})}}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1.16)$$

$$d_q^2 = d_a^2 + (d_a - d_{min})^2 [\frac{\Gamma(1+\frac{2}{c})}{\Gamma[(1+\frac{1}{c})^2 - 1][1 - (\frac{1}{n^{(1+\frac{1}{c})}})]^2}] \ldots\ldots (2.1.17)$$

To implement this general methodology, different tools are necessary for the construction of growth and yield models. They are briefly reviewed in the following section.

2.1.2 Tools

Without the necessary tools, growth and yield prediction can be difficult, impossible sometimes, to conduct. But those tools are seldom mentioned in the literature about growth and yield modelling, perhaps because they are assumed to be well known by researchers. They are mentioned here, to provide an indication of the value of those tools.

In nature, there are phenomena which take place randomly and which are thus not possible to represent in a fixed expression as can be done in some cases in mathematics and physics. Statistical techniques offer the best tool to deal with such phenomena. Growth and yield of forest crops are not entirely random phenomena but are influenced by a large number of inter-related environmental factors which change over time and in space. It is impossible to predict them by fixed relationships. They can only be described empirically using regression techniques and local data.

Regression techniques were not widely used in growth and yield modelling of forest crops until 1937, when MacKinney et al., constructed yield tables for non-normal loblolly pine stands using least-squares regression technique. Since then the techniques have become a major tool in forest growth and yield modelling (Ware et al., 1988).

Because of systematic correlation of growth data resulting from repeated measurement, certain efforts have been made to improve the precision of the least-squares techniques. For example, generalized least squares has been proposed for growth and yield modelling by Ferguson and Leech (1978) and further discussed by Davis and West (1981). However, these are still within the concept of least-squares and non-linear regression procedure.

With a large data set and without computers, regression techniques are difficult to apply. Computers revolutionised growth and yield modelling and made multiple regression or non-linear regression easy to conduct. High speed computers also made it possible to use individual trees as a basic unit of growth and yield modelling and to adopt alternative modelling approaches.

2.1.3 Modelling Alternatives

Analysis of growth and yield within this general framework can proceed using various modelling alternatives: stand-level only, diameter distribution, distance-dependent tree-level or distance-independent tree-level (Munro, 1974; Burkhart, 1977; Avery and Burkhart, 1983; Clutter et al., 1983).

The differences between stand-level and tree-level analysis is in the basic modelling units used: stand level models use per hectare values as the basic modelling units, while tree-level models use individual trees as the basic modelling units.

Models, regardless of the basic modelling units, may be deterministic or stochastic. In past applications, practically all stand-level models have been deterministic, while most tree-level models have involved some random elements.

Early stand-level models used graphical procedures (Lewis, 1954, Beekhuis, 1966). In later stand-level models, systems of equations were used and diameter distribution information generated as outputs in some cases (Bennett and Clutter, 1968; Lenhart and Clutter, 1971; Lenhart, 1972; Burkhart and Strub, 1974; Smalley and Bailey, 1974; Knoebel et al., 1986; Baldwin and Feduccia, 1987; Matney et al., 1987; Lenhart, 1988).

In distance-independent models the growth of individual trees is a function of variables such as present size, site index, and stand density. Tree mortality in these models may be predicted from past growth rates or generated randomly. Several approaches have been employed, such as that by Lemmon and Schumacher (1962), Stage (1973), Dale (1975), alder (1979), Belcher (1981), Manley (1981), Belcher et al., (1982).

In distance-dependent tree-level models, initial tree data are input or generated and each tree is assigned a co-ordinate location. Increment for each tree is calculated as a function of its size, site quality, and some function of distance to and size of neighbours which serves as an expression of competition. Growth is commonly adjusted by random elements representing genetic and/or microsite variability. Probability of tree death can be expressed as a function of competition and/or the individual tree's characteristics. Conventional stand and/or diameter distribution characteristics are tabulated and available as output at specified intervals. Models of this type have been produced by Newnham and Smith (1964), Arney (1974), Ek and Monserud (1974), Hegyi (1974), Lin (1974), Daniels and Burkhart (1975), Mitchell(1975), Daniels _et al_., (1979), Leary (1979), Tennent (1981), James (1983).

2.2 GROWTH AND YIELD MODELLING IN NEW ZEALAND

In New Zealand, yield tables prepared by Lewis (1954), and Spurr (1962, 1963) have played a major role in management planning during 1950's and early 1960's. From late 1960 onwards, more advanced computerised growth and yield models were used in forest management. In 1966, Beekhuis developed a growth and yield model for _Pinus radiata_ plantations. This together with Lewis' tables were the early variable density models used in New Zealand. Within its data range, the Beekuis model gave predictions of acceptable precision and it was employed for a long time being the only one widely available (Goulding, 1986). This model, however, is now considered to over-predict stands with intensive early thinning and pruning practices and low final stocking, which are common practice today in New Zealand (Goulding, 1986; Tennent, 1982).

Bailey produced a model for radiata pine plantations of N.Z. Forest Products Ltd in 1972 based on the Weibull distribution. The Weibull function has now been in widespread use for diameter distribution modelling since then. But the model itself fell into disfavour as it excluded heavier early thinning practices.

In 1974, Clutter and Allison devised a computerised growth simulator for N.Z. Forest Product Ltd. This model was valid for virtually any combination of thinning schedules and can provide yield predictions at both stand and size-class level (Whyte 1986). This model was subsequently revised by Woollons and Hayward (1985), and contains features which are of considerable interest to both researchers and managers.

KGM1, the Kaingaroa Growth Model for radiata pine, was developed by Elliott and Goulding in 1976. This model, together with its two successors, KGM2 and KGM3, and two similar models, AGM1 (for Auckland region) and SGM1 (for Southland region) have been extensively used by the former New Zealand Forest Service. These models (with the possible exception of KGM3) are considered to over-predict in stands which have sustained heavy early thinning.

West et al., (1982) constructed an early growth model for radiata pine. The model simulates a wide variety of silvicultural treatments on both forest and agricultural sites, for trees between the ages of 4 and 14 years and final stockings of 100 to 1000. A site or height/age curve and three different basal area increment functions were used to reflect regional variations.

Examples of individual tree models include Tennent's (1981) distance-dependent model which is said to predict adequately, although the mortality and diameter increment functions need re-examination (Dunningham and Lawrence 1987). Manley (1981) also produced a distance-independent model for radiata pine and its prediction is considered to be better than Tennent's or at least as good as Elliot and Goulding's stand model. James (1983) used a distance-dependent individual tree model to predict wood quality characters. None of these individual tree models, however, has been extensively used.

The stochastic differential equation model developed by Garcia (1978, 1984, 1987) has different mathematical features and has been attracting considerable professional interest throughout the world (Ware et al., 1988). Models of this kind have proved to be satisfactory to use in practice as well (Garcia, 1984; Dunningham and Lawrence, 1987). However, the differential equations, like any other equations, need also be subjected to biological and statistical tests when being fitted to data, and lack of goodness-of-fit in some of those equations has been experienced (Law, 1990). Furthermore, jointly estimating parameters of stand statistics projection equations can be unduly restrictive and so another approach widely used, which allows a wide range of equation forms to be evaluated and adopted if needed be. Garcia's approach, therefore, are not followed here.

2.3 GROWTH AND YIELD MODELLING OF DOUGLAS FIR

Growth and yield models for Douglas fir have been developed by, for example, Newnham and Smith (1964), Lin (1974), Bruce et al., (1977), Curtis et al., (1981) and others. Those models were developed for North America Douglas fir forests and would not be applicable to New Zealand Douglas fir plantations, because of environmental differences such as disease and insect influences and the differences in management practices.

In New Zealand, Douglas Fir has been largely ignored in terms of growth and yield modelling. Before DFCNIGM version 1 and version 2 were put into operation in 1989, the only computerized growth models available for the plantations were that for Kaingaroa forests by Mountfort (1978) and Nelson region by FRI (1979; see Law, 1990), both developed more than a decade ago. Before these early models, management of Douglas fir plantations relied on conventional yield tables prepared by Allsop (1949), Duff (1956), Spurr (1961, 1963) and Elliott (1969).

CHAPTER 3 DATA AND DATA ANALYSIS

3.1 REVIEW OF QUANTITY AND QUALITY OF THE GROWTH DATA

3.1.1 Quantity of the Data Set

The data were derived from permanent sample plot remeasurements in the form described by McEwen (1976). Procedures for taking measurements and making data entries are explained in detail in the Rotorua Conservancy PSP Manual and the Permanent Sample Plot Data Entry routines provided by the Forest Research Institute (FRI), Rotorua, New Zealand (Klitscher,1983).

Tables (3.1.1.1) and (3.1.1.2) show the distribution of measurements among forests, basal area/ha classes, age classes, initial stocking level and thinning operations.

Table (3.1.1.1), a summary of the whole data set used in this study, shows that:

(1) a total of 244 PSP plots, 2565 observations and 2320 increments (not shown on the table) are available for study;

(2) most of the measurements (75%) come from Kaingaroa forest.

The numbers of measurements available in Whakarewarewa, Whirinaki, Horohoro, Karioi and Pureora forests are 128, 70, 3, 125 and 107 respectively, with corresponding lower numbers of increments. These are insufficient, therefore, to develop separate models and their exclusion or inclusion

Table (3.1.1.1) Distribution of plot observations

conse-rvancy	forest	no of plots	age class	basal area class	stem/ha <200	201-400	401-600	601-800	801-1000	1001-1200	1201-1400	1401-1600	1601-1800	>1800	sum by B.A class	sum by age class	sum by forest
RO	WAKA	12	20	20						1			3		4		
				40					1						1	5	
			30	20						3					3		
				40				3	1						4	7	
			40	40		2	2	2							6		
				60			1	3							4	10	
			50	40	3	3									6		
				60		9	6								15		
				100					1	2		1			4	25	
			60	40	9										9		
				60	4	18									22		
				80		1	2								3		
				100					1						1	35	
			70	40	7										7		
				60	15	13									28		
				80		2									2	37	
			80	60	5	3									8	8	
			90	60	1										1	1	128
	KANG	163	10	20			1	1		2				2	6	6	
			20	20	4	134	37	48	8	8	13	12	1	2	267		
				40		6	15	64	12	26	33	42	1	25	224		
				60							2	1		13	16	507	
			30	20	12	54	12								78		
				40	17	108	66	53	4	16	14	6	4	4	292		
				60			14	31	14	15	18	23	13	34	162		
				80					1	2	3	5	11	5	27	559	
			40	40	7	28	14	1							50		
				60		22	25	7	1			1			56		
				80			6	5	2	4	2	3		1	23		
				100				1	9	9	2				21		
				120						1					1	151	
			50	20	3										3		
				40	26	16									42		
				60	9	90	37	4	4						144		
				80		20	46	13	5	8					92		
				100			4	15	17	6					42		
				120			2	4	6	7					19	342	
			60	40	37	28	3								68		
				60	37	123	7								167		
				80	1	20	22	1							44		
				100				10	5						15		
				120				2							2	296	
			70	40	11	1									12		
				60	7	21									28		
				80		4									4		
				100				1							1	45	
			80			1									1	1	1907

Table (3.1.1.1) (cont.) Distribution of plot observations

conse-rvancy	forest	no of plots	age class	basal area class	stem/ha <200	201-400	401-600	601-800	801-1000	1001-1200	1201-1400	1401-1600	1601-1800	>1800	sum by B.A class	sum by age class	sum by forest
	WAIM	21	20	40				1	2						3		
				60							1				1		
				80									1		1	5	
			30	40		1	3								4		
				60			3	2	2		2				9		
				80						4		1	1		6		
				100							1	1	1		3	22	
			40	40	2	16	3								21		
				60		12	31	2							45		
				80			13	4	2	2	2				23		
				100					7	1	5	4			17	106	
			50	40	11	10									21		
				60	3	22	6								31		
				80		3	17	2							22		
				100			4	1			1				6		
				120							1				1	81	81
			60	40	3										3		
				60	1	4									5		
				80		2									2		
				100			1								1	11	225
	WIRI	9	20	20			1								1		
				40			6								6		
				60										3	3	10	
			30	40			7			3			1		11		
				60			2	2	3			3	3	2	15		
				80			1				1	1	1	11	15	41	
			40	40			2								2		
				60		2	1	6	1						10		
				80				1		2	1		1	2	7	19	70
	HORO	1	30	20	3										3	3	3
WN	KROI	21	30	40			1								1		
				60			1	2	1	1					5		
				80							1				1		
				100							1				1	8	
			40	40		1									1		
				60		4	1	1	2	3					11		
				80		1		1							2		
				100							1				1		
				120						1	2				3	18	
			50	60	2	2									4		
				80		6	1	3	2						12		
				100		1	5		1						7		
				120			3	2	12	6					23		
				140				4	14	10					28		
				160					1	3					4	78	
			60	100			6								6		
				120			1								1		

Table (3.1.1.1) (cont.) Distribution of plot observations

conse-rvancy	forest	no of plots	age class	basal area class	stem/ha <200	201-400	401-600	601-800	801-1000	1001-1200	1201-1400	1401-1600	1601-1800	>1800	sum by B.A class	sum by age class	sum by forest
				140				2	3						5		
				160			1	1	6	1					9	21	125
AK	PURE	17	10	20										1	1	1	
			20	20		10	2	7					1		20		
				40						1		2	4	3	10		
				60							1			1	2	32	
			30	20		8	1	1							10		
				40		3	11	4							18		
				60			4	3	3	4	1		3		18		
				80							2	3	1	3	9	55	
			40	40		11	2								13		
				60			1	1							2		
				80					2		1			1	4	19	107
sum		244			240	846	464	322	156	150	112	111	48	116	2565	2565	2565
% of total observations					9.4	33	18.1	12.6	6.1	5.9	4.4	4.3	1.9	4.5			

in the Kaingaroa group will be judged in terms of other factors such as location, soil types and consistency of growth pattern within the main body of data.

(3) there are only a few measurements for age classes 10 and 80, so the age range in the data set can be said to be lie predominantly between 15 and 70 years, for all practical purposes;

Table (3.1.1.2) Summary of thinning operations

thinnings	0	1st	2nd	3rd	4th
number of plots	64	133	29	17	1
no. of observations	693	1331	219	72	4
maximum N/ha removed	0	2599	1503	336	128
minimum N/ha removed	0	25	10	20	128
mean N/ha removed	0	846	269	164	128
mean residual N/ha	1150	559	290	225	148
Maximum thinning intervals (years)	0	11	6	4	
Minimum thinning intervals (years)	0	2	2	4	
Mean thinning intervals (years)	0	5	4	4	

(4) an adequate range of stocking levels is represented

within the data set, particularly for Kaingaroa, Waimihia, Whirinaki and Pureora forests.

Thinning operations are summarized in Table (3.1.1.2). In the data set, there are some plots left unthinned and others thinned up to four times (one plot only). On average the plots were thinned three time at ages around 25, 30 and 40 years with a final stocking of 560, 290 and 230 / ha respectively. Time intervals between thinnings average 5 years between first and second and 4 years between the second and third.

3.1.2 Quality of the Growth Data

The growth data are of good quality for modelling purposes:

(1) a total of 2565 available measurements represents more than an adequate quantity;

(2) there is a relatively large number of measurements per plot, the minimum number of measurements per plot is 3, maximum 18 and an average of 10;

(3) wide ranges of age, initial stocking and thinning operations are covered.

There is room for improvement, however: for example, there are no records about disease infection; individual tree information is insufficient for developing individual tree equations; there are no record of merchantable volume; and there are measurement errors.

Errors were analyzed through fitting equations to the data, isolating observations that had a residual of more than 3.5 times of the standard deviation (outliers) and then printing them out. The original data records were then checked. These observations were corrected, if possible, from such objective evidence, but if not available, the data were provisionally accepted. Some of these observations with inexplicable errors were later excluded when fitting the equations.

3.2 SOURCES OF VARIATIONS

The following factors were expected to affect growth and thus had to be accommodated in the model fitting process:

(1) disease infection;

(2) locality;

(3) altitude;

(4) spacing;

(5) thinning;

(6) growth period.

Each factor was thoroughly considered and the results of the analyses are reported in the following subsections.

3.2.1 Disease Infection

Douglas fir plantations are now largely infected by a needle-cast fungus, Phaeocryptopus gaeumannii. Examination of the residual patterns of basal area projection equation (Fig 4.4.3.1) shows that discordant patterns exist and these might be due to disease infection. Thus how to model diseased and undiseased stands became a major component of this study. It was originally envisaged that this could be done in one of the following ways:

a. disease infection classes were formed and the infection level expressed as a variable in the yield equations;

b. separate equations were fitted for each distinct infection level;

c. a disease index was derived from recorded information and used in the way described in a or b; or

d. yield equations were fitted to diseased and healthy crop data to the extent that time of infection was known to have occurred.

Approaches a, b and c needed information about the disease on a plot or individual tree basis; unfortunately, it was subsequently found that such comprehensive information is not available.

There is no disease information in the original data set provided by Timberlands Ltd. Later, some descriptions about the disease were extracted from FRI sample plot records but only 35 out of 244 plots have such descriptions, a number which proved to be insufficient to draw any well founded conclusion. Those plots are: 28 in Kaingaroa, 4 in Waimihia, 3 in Karioi and none for the rest of the forests.

Even for plots with disease information extracted, the descriptions are too simple to be of any use. For example, the whole description for Kaingaroa experiment 906/0/3/0 in compartment 1365 states: "8/1982, canopy closure 95%, crowns still healthy".

There is a possibility that the discordant patterns on the residual graphs (section 4.4.3.3) may have been more or less caused by differences among seed origins of the plots. Thus seed origins were examined. A series of maps of seed origins, provided by Timberlands Ltd, indicated only compartment and seedlot number, and so there can be no certainty about the seed origin of a particular seedlot number. Thus, this approach could not proceed further.

A number of studies carried out in the past (Beekhuis, 1978; James and Bunn, 1978; Manley, 1985) indicated that there was no obvious growth loss due to the disease before 1963. Thus approach c was tried in this study using 1963 as the threshold for disease occurrence and basal area per hectare as an index of response subject to local divergences. Symbolically, $d_x = (G_u - G_d)/G_u$, where, d_x is the disease index, G_u and G_d represent basal area/ha for undiseased stand (pre-1963) and diseased stand (post-1963) respectively, was calculated for each plot. Then the Schumacher basal area equation was fitted to the data with d_x expressed as one of the independent variables. But no obvious improvement resulted from this attempt.

Approach d was the one finally adopted using 1963 as the threshold for disease occurrence just described above.

Results showed considerable yield difference between diseased and undiseased stands. This indicates that it is mainly the disease that caused growth loss. But the discordant patterns (Fig 4.4.3.1) cannot be eliminated by using such an approach because: a certain number of plots may not have been infected by the disease even after 1963, which means that there are still some plots in the post-63 group that have grown at a normal rate after 1963. When these normal growing plot measurements are mixed with ones having reduced growth-rate, the residual graphs of equations fitted to the mixed data set will show discordant patterns. The reverse may be true for those plots established around the Taupo area, where the disease was first found.

From Auckland down to Timaru (possibly even further south) all Douglas fir plantations have now been infected by the disease (Hood and Kershaw, 1975). Thus the aim of employing this approach was to provide a way for managers and those interested in the management of the plantations to use local information on presence or absence of infection to best effect.

3.2.2 Locality

Data for this study come from 7 forests in the Central North Island. Should models be developed on the basis of whole data set or some kind of grouping of the whole data set should be made? This will be described in this section.

In the ensuing analysis, the following colours and numbers refer to single forests

Table (3.2.2.1) Forest reference number and colour

Forest	Number	Colour
Horohoro	1	red
Kaingaroa	2	black
Karioi	3	blue
Pureora	4	green
Waimihia	5	orange
Whakarewarewa	6	purple
Whirinaki	7	brown

In even aged stands, mean top height is little affected by density and intermediate cutting, except through thinning from above. Mean top height is commonly used as a indicator of site quality (see section 4.4.2.1) which is synonymous with locality in terms of growth of mean top height. Three groups of localities were finally formed from analysis of equations based on mean top height.

The three groups finally formed are:

a. Whakarewarewa;

b. Karioi;

c. Kaingaroa, Waimihia, Whirinaki and Pureora.

The reasoning behind such grouping is set out below. Among the 7 forests, there were only two pairs of observations for Horohoro forest and so this forest was removed altogether from consideration.

A graph of mean top height against age [Fig (3.2.2.1)] indicated that all forests could be combined for height growth trends. Observations for all forests appeared to be spread over the graph evenly and no separate pattern was discernible from a purely visual analysis.

Non-linear regression analysis was also performed on the whole data set for mean top height ($h_{100,T}$), however, using the Chapman-Richards equation (The Chapman-Richards equation was used since it produced the best fit among others tried). It fitted very well according to the pattern of residuals [Fig (3.2.2.2)], implying the same consistency

$$h_{100,2}=h_{100,1}\left[\frac{1-\exp(-\alpha T_2)}{1-\exp(-\alpha T_1)}\right]^{\beta} \quad \text{(3.2.2.1)}$$

as for Fig (3.2.2.1).

PLOT OF MEAN TOP HEIGHT ON AGE

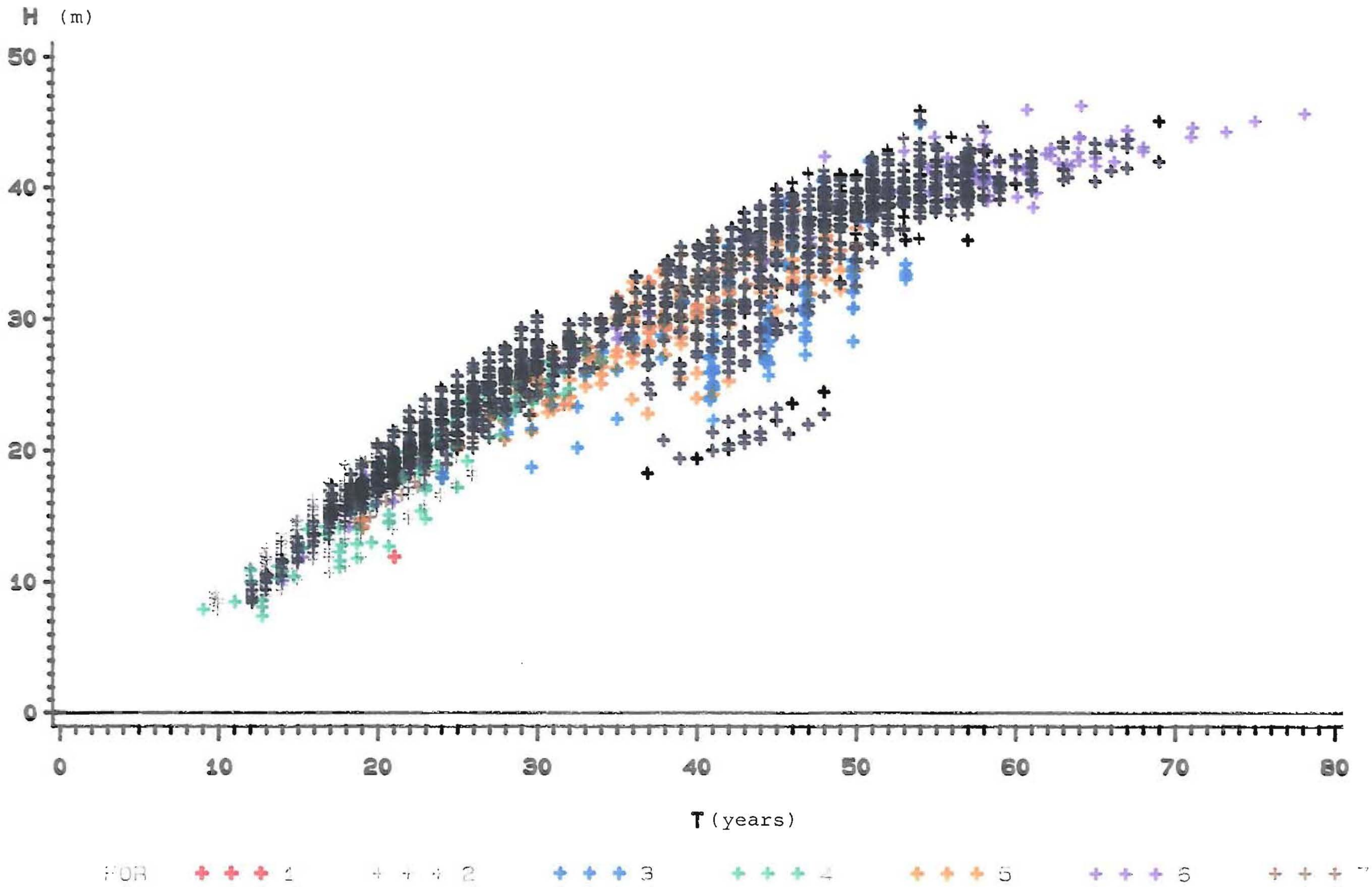


FIG (3.2.2.1): CENTRAL NORTH ISLAND PLANTATIONS

RESIDUAL PLOT OF SITE INDEX EQUATION

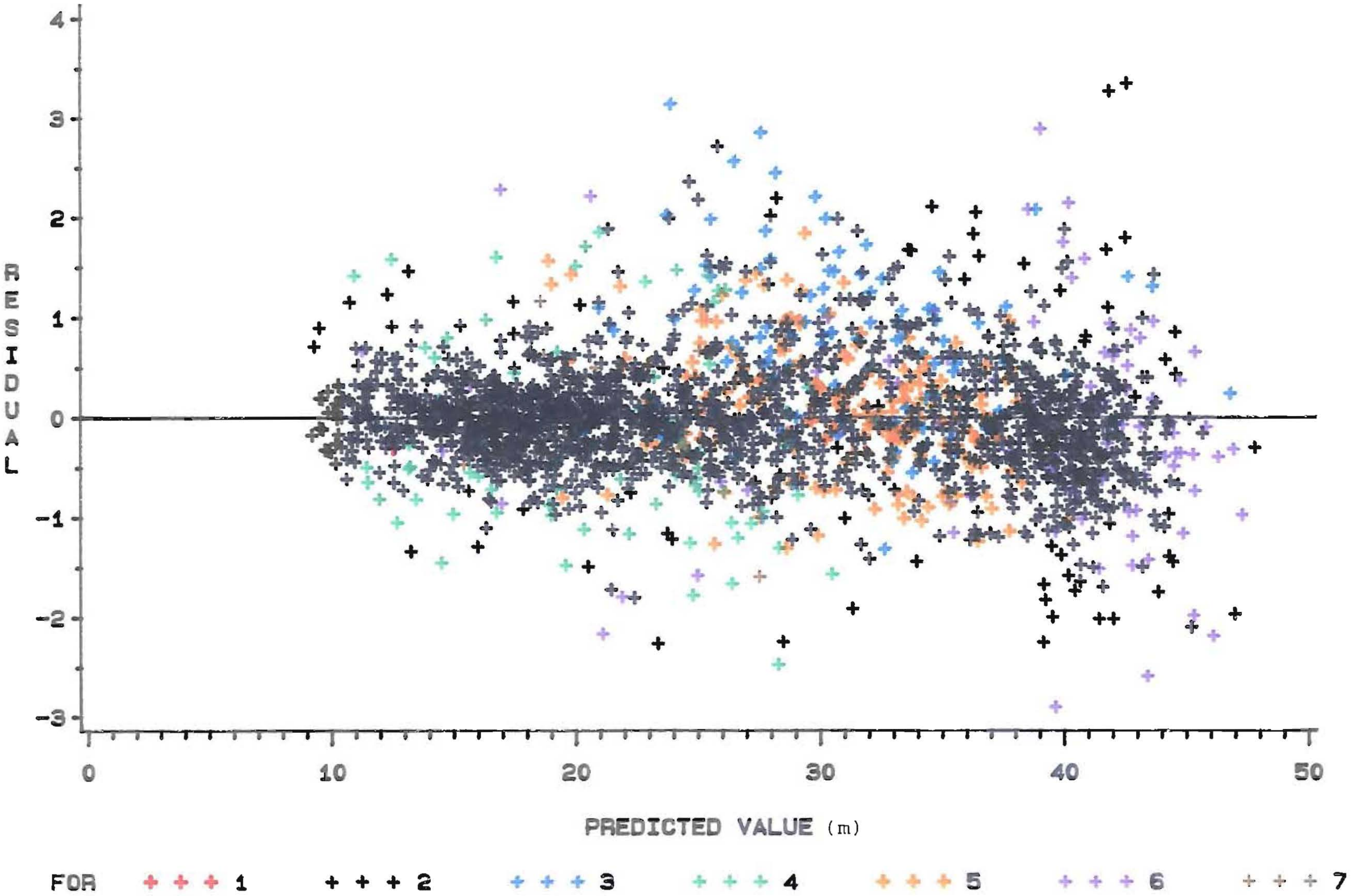


FIG (3.2.2.2): CRNTRAL NORTH ISLAND PLANTATIONS

The same equation was next fitted to each group separately. Table (3.2.2.1) contains statistics of the equation for each group.

Table (3.2.2.1) Coefficients of the height equations

For	Estimates of α	Estimates of β	STD error of α	STD error of β	ess	n
KANG	0.0353	1.6202	0.0012	0.0380	0.3045	1867
WAKA	0.03122	1.2872	0.0079	0.2876	0.8430	93
KROI	0.01607	1.4589	0.0065	0.2262	0.5461	100
PURE	0.0372	1.5154	0.0105	0.1938	0.7983	81

It can be seen that coefficients for Whakarewarewa and Karioi are distinct from those of the Kaingaroa group (Kaingaroa, Pureora, Waimihi and Whirinaki), So Whakarewarewa and Karioi were kept separate from the Kaingaroa group. This analysis seemed to be an indication that graphs depict general patterns. To do more sensitive analysis, numerical test has to be carried out rather than graphical analysis alone. More explanations about this are given in section 4.1.4. of Chapter 4.

The numbers of measurements available for Whakarewarewa and Karioi are 93 and 100 respectively, too small to reflect reliably the intrinsic growth of those forests and to justify their inclusion in the main data set.

Pureora is separated geographically from Kaingaroa forest, but coefficients for the equations indicated that these two groups could be safely combined. If there were sufficiently large numbers of measurements for Pureora forest (a total of only 81 is currently available), that conclusion could need modification. To some extent, therefore, caution should be exercised when the model is applied to Pureora. The above grouping has also been suggested independently by Tennent (pers. comm.).

Although localities were finally divided into three groups, there were sufficient numbers of observations only for the Kaingaroa group to fit reliable growth and yield equations.

3.2.3 Altitude

Table (3.2.3.1) shows the average of mean top height derived from the data summarised by age and altitude classes. Evidently, mean top height does not vary much with altitude below 700 m. For altitude over 700 m, mean top height decreases with altitude but not to any great extent. This would be more obvious if age class intervals were reduced to 5 or fewer rather than 10 because of the age effects. For example, mean top height obviously decreases with altitude for age class 60 at an altitude of over 700 m; this might be a result of younger trees within the class growing at these higher altitudes. Plot measurements not distributed evenly among altitude classes may also be a reason for such a trend.

Woollons et al., (1985) used altitude as an independent variable in a site index equation for radiata pine in Kinleith Forest, also in the Central North Island of New Zealand. Altitude has also been tried in this study to see if it is a useful independent variable in the site index equation. For the Whakarewarewa and Karioi groups, predicted mean top height increases with altitude when altitude is introduced into the site index equation as an independent variable. For the Kaingaroa group, the equation gives a more useful prediction, i.e. predicted mean top height decreases with altitude, but the residual sum of squares was reduced by only 1.7% and it was not reduced at all when the equation was fitted to the whole data set. Thus, it was concluded that the influence of altitude is not significant for the range of altitudes covered by this data set and altitude was not included in the best fitting site index equations finally adopted.

Table (3.2.3.1) Mean top height by age and altitude classes

age class	Altitude class ≤200	300	400	500	600	700	800	>900
10				8.9	7.9			
20		14.6	16.3	13.3	12.5	12.5	10.9	
30		19.8	22.5	19.3	21.9	19.6	20.2	20.0
40		29.2	28.5	29.7	28.9	28.6	26.8	27.6
50	37.2	36.7	37.7	37.3	35.8	31.1	30.2	30.0
60	38.0	39.3	41.4	40.3	39.3	38.4	35.8	33.4
70		40.9	46.2	42.3	41.7			
80		44.9	44.6					

3.2.4 Spacing

There were six classes of initial spacing in the data set as shown in Table (3.2.4.1). Differences among initial stems/ha levels were tested by fitting a basal area equation, in this case the Schumacher form (equation 3.2.4.1), to each initial stocking level. The asymptotic standard errors were then compared. The Schumacher equation was used because it resulted in a very good fit for basal area projection and because it converged easily.

The numbers of observations in classes 2778, 4444 and

6944 were too small to fit separate equations. Classes 2315, 2778 and 3086 can be combined as regression coefficients of these two classes were nearly identical. Thus two main groups were considered: (a) 1736 and (b) 2315 to 3086. Eventually, all initial stocking levels were combined (c) because standard errors of the combined group were lower than those of the two separately fitted groups (Table 3.2.4.2). Furthermore, it was hoped that predictive flexibility could also be increased if the models were fitted to all initial spacing regimes combined.

Table (3.2.4.1) Distribution of measurements by initial stocking and forests

For.	**Initial stocking level**						
	1736	2315	2778	3086	4444	6944	total
HORO	0	0	0	2	0	0	2
KANG	579	392	0	771	0	1	1743
KROI	104	0	0	0	0	0	104
PURE	0	0	0	80	10	0	90
WAIM	203	0	0	0	0	0	203
WAKA	11	0	0	63	0	42	116
WIRI	0	0	17	44	0	0	61

Table (3.2.4.2) Comparison of initial stocking level

$\log(G_2) = \log(G_1)(T_1/T_2)^{\beta} + \alpha(1-(T_1/T_2)^{\beta})$ .....(3.2.4.1)					
set	**estimates of**		**STD error of**		**n**
	α	β	α	β	
1736	5.7733959	0.5219554	0.2115409	0.0587559	911
2315-3086	4.6342635	1.2658073	0.0212909	0.0216382	287
comb-ined	4.733519	1.1701647	0.0196016	0.0192757	1201

3.2.5 Thinning

During the process of developing DFCNIGM1 and DFCNIGM2, separate basal area equations were fitted for unthinned stands, thinned once, thinned twice and thinned a third time. The equation used was the Schumacher. Table (3.2.5.1) shows the coefficients of the equation fitted for each group. There was little difference between coefficients for unthinned or one thinning only, but there were with more than one thinning. That latter group contained too few data, however, and so separation into thinned and unthinned was made.

On the other hand, the possibility of fitting one equation to thinned and unthinned pooled data was examined during developing of DFCNIMG3. In the literature, there are examples of treating thinning by fitting separate equations for each thinning regime (Baldwin and Feduccia. 1987; Bailey et al 1981; Matney and Sullivan, 1982). There are also studies which develop equations to predict stand statistics after thinning and then use the after thinning statistics as another starting point for further projection (Knoebel et al., 1986). Cao and Burkhart (1984) used a segmented approach to model thinned stands.

As distinct from DFCNIGM1 and DFCNIGM2, thinning effects in DFCNIGM3 were approached by pooling the thinned and unthinned data and introducing a thinning index to the equations, then fitting them to the pooled data.

Thinnings affect stand mean diameter, the first moment of the initial distribution. This was considered by introducing the thinning index into some of the Reverse Weibull parameter projection equations. There are examples of using thinning index in yield equations (e.g. Pienaar, 1979; Clutter and Jones, 1980; Bailey and Ware, 1983; Pienaar et al., 1985; Pienaar and Shiver, 1986; Murphy and Farrar 1988). But it seems that such an index has not been previously used in parameter projection equations.

Table (3.2.5.1) Comparison of thinning regimes

thinnings	estimates of		STD error	lower	upper	n
				asymptotes		
0	α	5.013515	0.003207	4.950495	5.076535	451
	ß	0.913518	0.030118	0.854327	0.972709	
1st	α	5.094112	0.039819	5.015981	5.172243	1117
	ß	0.925192	0.023233	0.879605	0.970778	
2nd	α	7.944781	2.168825	3.660029	12.229531	154
	ß	0.277587	0.148315	-0.015426	0.570600	
3rd	α	8.506121	4.568573	-0.653307	17.665550	55
	ß	0.224252	0.219989	-0.216799	0.665303	
4th	α	4.687444	0.427290	3.677055	5.697832	8
	ß	0.900876	0.463699	-0.195609	1.997361	
2nd-4th	α	7.239693	1.334872	4.608755	9.870632	217
	ß	0.315333	0.124774	0.069413	0.561254	
0th-4th	α	5.094091	0.038693	5.018185	5.169998	1334
	ß	0.910657	0.013612	0.864334	0.956978	

3.2.6 Growth periods

There are studies, where yield equations were fitted for each distinct growth period, as judged from their parameters or other statistics (Mountfort, 1978; Knoebel et al., 1986), but this was not followed here because:

(a) if different equations were fitted to different growth periods of the same plot, the curves drawn from two adjacent equations will not necessarily join together to form a smooth sigmoid curve a plantation population should follow;

(b) ages have been expressed in all key projection equations and the effect of growth period on yield will be reflected by those variables.

(c) however, the Schumacher equation was fitted to basal area data for 10 year growth intervals (i.e., T_1 to T_2) to see if there are great differences among growth periods. Table (3.2.6.1) contains the statistics of those equations. As can be seen from the table, the differences among the coefficients for all of the intervals are not great except ß value for the growth period $50 < T \leq 60$ while the number of observations in intervals of $T \leq 10$ and $T > 70$ are too small to warrant any conclusion.

Table (3.2.6.1) Statistics of basal area equation for different growth periods

Intervals		Estimates	STD Errors	Confid. intervals Lower	Upper	n
$0 < T \leq 10$	α	4.43586	0.12614	4.12720	4.74452	7
	β	1.40373	0.12414	1.09996	1.70750	
$10 < T \leq 20$	α	4.90324	0.03677	4.83099	4.97548	520
	β	1.00450	0.02694	0.95158	1.05742	
$20 < T \leq 30$	α	4.83180	0.03255	4.76785	4.89576	529
	β	1.12723	0.03594	1.05664	1.19783	
$30 < T \leq 40$	α	5.13215	0.08075	4.97265	5.29164	159
	β	0.92183	0.08073	0.76238	1.08128	
$40 < T \leq 50$	α	5.04950	0.06266	4.92632	5.17287	319
	β	0.87578	0.05946	0.75879	0.99278	
$50 < T \leq 60$	α	5.55236	0.21413	5.13036	5.97435	223
	β	0.48970	0.06833	0.35504	0.62436	
$60 < T \leq 70$	α	7.63929	2.33725	2.82567	12.4529	26
	β	0.27166	0.17136	-0.08126	0.62457	

3.3 CONCLUSIONS REGARDING THE SCREENING OF DATA AND THEIR SOURCES OF VARIATIONS

(1) In terms of disease, data are simply divided into two groups: before and after 1963, at which latter time the

disease was clearly established in the region. There is no clear evidence to justify further breakdown despite much wider investigation;

(2) Localities are divided into three groups: Whakarewarewa, Karioi and the Kaingaroa (including Pureora), but only the Kaingaroa group has a large enough data base to be modelled specifically;

(3) For this data set, the effect of altitude is not readily apparent;

(4) All initial stocking levels can be safely combined;

(5) Thinned and unthinned stands were modelled separately in DFCNIGM1 and DFCNIGM2, but jointly in DFCNIGM3 through introducing a thinning index;

(6) Modelling has to be re-initialised after any thinning, but showed no inconsistency between any changes in density irrespective of the length of time between which they occurred.

CHAPTER 4 STAND GROWTH AND YIELD MODELS DFCNIGM

4.1 MODELLING METHODOLOGY

4.1.1 Creation of Data Sets

Originally, values for 23 derived variables were extracted from PSP raw data and four derived variables from alignment charts constructed by Lewis (1954). These 27 variables are: experiment (expt), forest (for), altitude (alt), stems/ha (stok), number of thinnings (c), age of measurements T_1 and T_2 (where the subscripts 1 and 2 represent beginning and end of a growth period and apply to all variables having one of them), gross basal area/ha (G_{g1}, G_{g2}), net basal area/ha (G_{n1}, G_{n2}), gross stems/ha (N_{g1}, N_{g2}), net stems/ha (N_{n1}, N_{n2}), gross volume/ha (V_{g1}, V_{g2}), net volume/ha (V_{n1}, V_{n2}), gross merchantable volume/ha (V_{mg1}, V_{mg2}), net merchantable volume/ha (V_{mn1}, V_{mn2}), gross dbh (d_{gg}), net dbh (d_{gn}), mean top height (h_{100}) and site index (S). The values of these variables were punched onto the VAX system under the file name DAT.ORGN and the original data file presented fully in Appendix 7 was formed.

4.1.2 Data Format

The original file DAT.ORGN was in a list format. When fitting equations to a data set, it is often necessary to change the equation form and the order in which the data are read by statistical packages such as SAS. As list formatted data can only be read in sequence, a separate file was then prepared for each model using the column format so that different equation forms and variables could be tried. The file names and the variables in the files for each model are listed below. If a new variable was needed for fitting an equation that was not in the original file and could not be derived from the existing data, it was input manually on the keyboard. The following files can be found in diskette form in Appendix 7.

Site index equation____ file is SD3.SAS, containing variables: plot, for, T_1, T_2, $h_{100,1}$, $h_{100,2}$, alt, S_p, S_s.

Basal area/ha equation______ file is BANEW35.SAS, containing plot, for, T_1, T_2, G_{n1}, G_{n2}, N_{n1}, N_{n2}, d_{gn1}, d_{gn2}, d_b, d_a, d_t, S, c, T, year.

Volume equations______ file is VT35.SAS, containing variables plot, for , T_1, T_2, V_{n1}, V_{n2}, V_{mn1}, V_{mn2}, G_{n1}, G_{n2}, $h_{100,1}$, $h_{100,2}$, c, year, G_{r1}, G_{r2}, T.

Merchantable volume equation____ file is VMR35.SAS, containing variables: plot, for, T_1, T_2, V_{n1}, V_{n2}, V_{mn1}, V_{mn2}, G_{n1}, G_{n2}, $h_{100,1}$, $h_{100,2}$, c, year, d_{gn1}, d_{gn2}, T.

Mortality equation______ file is M.SAS, containing variables: plot, for, T_1, T_2, G_{n1}, G_{n2}, N_{n1}, N_{n2}, d_{gn1}, d_{gn2}, d_b, d_a, d_t, S, $h_{100,1}$, $h_{100,2}$, year.

Weibull parameter projection equations_____ file is DA4.SAS, containing variables: plot, for, T_1, T_2, d_{max1}, d_{max2}, d_{min1}, d_{min2}, d_{mean1}, d_{mean2}, d_{std1}, d_{std2}, d_b, d_a, d_t, d_{gn1}, d_{gn2}, N_{n1}.

Extreme value projection equations: the file is DMAX.SAS, containing the same variables as in file DA4.SAS.

Height corresponding to mid-point diameter class__ file is HDR.SAS, containing variables: plot, for, year, tno, h, dbh, T, d_{100}, h_{100}, N, G, V.

Symbols representing variables that are not in the original data set, DAT.ORGN, are defined below.

plot____ plot number;

$h_{100,1}$ $h_{100,2}$ ____ mean top height;

S_p S_s____ site index predicted by the new site index equation and Schumacher equation;

d_a, d_b, d_t____ mean dbh of the stand after and before thinning and of the trees thinned;

T____Time of thinning;

year____year in which the measurement was taken;

G_{r1}, G_{r2}____basal area/ha removed in thinning;

d_{max1}, d_{max2} ____ maximum diameter of the plot;

d_{min1}, d_{min2}____minimum diameter of the plot;

d_{mean1}, d_{mean2}____ arithmetic mean diameter of the plot;

d_{std1}, d_{std2}____standard deviation of the diameters;

tno____index number of a tree in a plot;

h____ height of a tree in a plot;

dbh____dbhob of tree in a plot;

T_t____age of tree in a plot;

d_{100}____mean top dbhob (i.e. average dbhob of the largest 100 trees/ha);

N, G, V___ stems/ha, basal area/ha and volume/ha respectively.

4.1.3 Checking of Reliability of Data

The reliability of the data was checked by isolating the observations having a residual more than 3.5 times of the standard deviation (outliers) and printing them out using the following SAS code (after MODEL statement in PROC NLIN or PROC REG procedure):

```
OUTPUT OUT=STATS R=RESID P=PRED;
ID _SSE_ _PLOT_;
DATA X;
SET STATS;
RMS = SQRT (_SSE_/n);
TEST = ABS (RESID);
RND = TEST/RMS;
IF RND LT 3.5 THEN DELETE;
PROC PRINT DATA = X;
VAR PLOT FOR T1 T2 RND;
```

Then the original data records corresponding to the print-out variables were checked. A measurement error was corrected wherever feasible or, if not, the observation was reinstated as is. The observations were ignored in most cases because it is not known what caused the errors, say 30 years ago, when the measurement was taken. This emphasises the importance of having reliable measurements and conducting rigorous data checks during the initial processing of information for growth studies.

4.1.4 Modelling Approach

The major technical procedures used in the study are non-linear regression, ordinary least-squares regression, analysis of variance, and univariate analysis. These procedures were utilized in the form of the procedures PROC NLIN, PROC REG, PROC GLM, PROC UNIVARIATE and PROC MEANS on the SAS package (SAS institute Inc., NC, USA. 1985).

Regression equations can be fitted to a data set of any sort, but this study also tried to ensure that:

(1) the variables in the data set conform to biologically appropriate and mathematically sound relationships;

(2) the functions used are of an appropriate form to represent the intended relationships;

(3) a good fit is produced.

In forest yield research, it has been a tradition to install permanent sample plots and take repeated measurements in them. Several variables are included in the measurements so that all important stand statistics can be analyzed. Well established functional forms for characterising variable development over time are available for such an analysis; for example, the Schumacher basal area equation (Clutter, 1963), the Chapman-Richards height equation and the combined variable volume equation (Clutter et al., 1983). It has been pointed out by Woollons et al., (1990) that various equations may be appropriate to any one circumstance. The nature of the data, therefore, may well determine which is best in each case.

Usually, after fitting an equation to the data, the goodness of fit of the equation (linear or nonlinear) is then judged and compared by analyzing the standard error of parameter estimates, confidence intervals, coefficients of variation and, to some extent graphical residual patterns of the equations. When analyzing permanent sample plot data, the residual sums of squares are under-estimated because the observations are systematically correlated by virtue of repeated measurements on the same sets of trees. In this study, the fit of an equation is judged by simultaneously performing several analysis, namely, analysis of the standard error of estimates,confidence intervals, coefficients of variation, residual patterns, mean of residuals, extreme values of residuals and univariate analysis.

It is found that the residual chart may be a somewhat biased approach depending on the choice of class intervals. The chart is a graphical presentation of the frequency distribution of the residuals. It is based on the assumption that the residuals are normally distributed provided that the regression equation involved is a good fit. This is used in reverse order to check the fit of the equation: i.e. if the residuals are normally distributed then we conclude that the equation is unbiased and if tightly dispersed around the 0 reference line, then it is of good fit.

However, for a given data set and a fitted equation, different scales on the horizontal axis of the residual bar chart will change the shape of the chart. If the scale is large, there might be little bias shown and if the scale became small, bias will start to appear. In other words the degree of bias shown is dependent on an arbitrarily chosen scale. To circumvent this problem, some other procedures were used to check the distribution of the residuals further.

(1) The univariate analysis procedure

In univariate analysis, several statistics can be calculated. For example, the Kolomogorov D statistic can be computed and a normality test conducted. If the residuals are normally distributed then the equation has produced a good fit otherwise its fit need re-examination.

The skewness coefficient, $\beta_1^{1/2}$, and kurtosis coefficient, β_2, were also used to further check the residual distribution.

$$\beta_1^{\frac{1}{2}}=\frac{\mu_3}{\mu_2^{\frac{3}{2}}}\qquad (4.1.4.1)$$

$$\beta_2=\frac{\mu_4}{\mu_2^2}\qquad (4.1.4.2)$$

$$\mu_K=\int[x-E(x)]^K f(x)\,dx\qquad (4.1.4.3)$$

Where f(x), E(x) are the probability density function, in this case the normal distribution function, and expectation of the random variable x, the residuals. K is a constant.

The skewness, or asymmetry, is defined as a departure from symmetry about the mean where negative values indicate a distribution with a long tail to the left and positive values a long tail to the right. Kurtosis is a relative measure of the flatness or degree of peaking of a distribution. The larger the value of β_2 the more peaked the distribution (Hafley and Schreuder 1977).

(2) The extremes, mean and absolute mean of residuals

Other statistics used to judge the fit of an equation are the mean residual, the absolute mean and extremes of the residuals.

The mean of the residual must be zero or close to it if an equation is to produce an unbiased fit. The absolute mean residual indicates average prediction error. The two extremes, maximum and minimum, represent the maximum predicted error. If a basal area equation, for instance, produces a good fit in terms of residual pattern, but the absolute extreme is more than, say 20 m^2/ha, then re-examination of the fit of the equation should be carried out. For a model to produce a good fit, its asymptotic standard errors should be less than 10 percent of the estimate, the residual patterns and residual bar charts show no or little bias and the residuals are normally distributed with little or no skewness; the mean of the residuals should lie within 1% around 0; the absolute mean and the extremes are within a range that is reasonable for the particular variable being modelled. For example, a maximum predicted error of 20 m^2/ha for a basal area projection equation would be too high.

4.2 GROWTH AND YIELD MODEL DFCNIGM1

DFCNIGM1 (Appendix 4), the acronym for Douglas Fir Central North Island Growth Model version 1, is a whole stand model applicable to Douglas fir plantations in Kaingaroa, Waimihia, Pureora and Whirinaki forests in the Central North Island of New Zealand. Details of DFCNIGM1 were given in a report, Douglas Fir Central North Island Growth Model (Liu Xu, 1989), to the New Zealand Forestry Cooperation.

Equation forms of DFCNIGM1 are all the same as that of stand level components of DFCNIGM3 given in section 4.4 of this chapter except basal area and mortality equation. Coefficients of all the equations in the two models are different.

4.3 GROWTH AND YIELD MODEL DFCNIGM2

DFCNIGM1 was completed and put into operation in March 1989. Users' reaction to the model was that it gave good prediction for all stand statistics except mortality, which seemed to be slightly over-predicted. DFCNIGM1 was then revised to produce DFCNIGM2, Douglas Fir Central North Island Growth Model version 2 with an improved mortality equation and with the Hossfeld equation in place of the

Schumacher basal area equation. The format for running DFCNIGM2 is also different from that of DFCNIGM1. Details in terms of site index equation, basal area projection equations, volume equations, merchantable volume equations and mortality equations of DFCNIGM2 are presented in FRI/Industry Research STAND GROWTH MODELLING COOPERATIVE report No. 17 (Liu Xu, 1990. see Appendix 2).

All equation forms of DFCNIGM2 are the same as stand level model components of DFCNIGM3, details of which will be given later in this chapter.

4.4 GROWTH AND YIELD MODEL DFCNIGM3 ___Stand Level Model Components

DFCNGIM3 consists of a stand level model and a diameter distribution model. This section describes the stand level model components of DFCNIGM3 and the diameter distribution model components are presented in chapter 5.

4.4.1 Equations Employed for Stand Level Model of DFCNIGM3

Equations employed in stand level model of DFCNIGM3, which are the same as that used in DFCNIGM2, are listed below

Site index equation

$$S=h_{100,1}\left[\frac{1-\exp(-\alpha T_0)}{1-\exp(-\alpha T_1)}\right]^{\beta} \qquad (4.4.1.1)$$

Basal area projection equation

$$Y_2=\frac{1}{\left(\frac{1}{Y_1}\right)\left(\frac{T_1}{T_2}\right)^{\gamma}+\left(\frac{1}{\alpha}\right)\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma}\right)+X\left(\frac{1}{\beta}\right)\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma}\right)} \qquad (4.4.1.2)$$

Total volume equation, diseased stands

$$V = \alpha+\beta G+\gamma h_{100}+\delta G h_{100} \qquad (4.4.1.3)$$

Total volume equation, disease-free stands

$$V_t = \beta h_{100} + \gamma G + \delta h_{100} G \qquad (4.4.1.4)$$

Merchantable volume equation

$$V_m=\alpha V_t^{\beta}\exp\left[-\gamma\left(\frac{15}{d^{\delta}}\right)\right] \qquad (4.4.1.5)$$

Mortality equation

$$N_2=N_1\left(\frac{T_2}{T_1}\right)^{\beta_2 (T_2-T_1)(\beta_1 G+\beta_3 S+\beta_4 d_g) X} e^{(T_2-T_1)(\beta_1 G+\beta_3 S+\beta_4 d_g) X} \qquad (4.4.1.6)$$

The specific equations employed in the model will be described in more detail later in this chapter.

4.4.2 Site Index Equations

4.4.2.1 An overview

Site quality can be evaluated in several ways. It can be evaluated indirectly by using inter-species relationships. For example, Coile (1948) used this method to calculate site index for loblolly pine and shortleaf pine. Olson and Della-Bianca (1959) used such an approach for mixed species stands.

Site quality can also be evaluated through using the characteristics of other vegetation on the site. This approach is based on the fact that both the species of interest and other species in the stand are influenced by environmental factors. The presence or absence or vigour of other species could provide information on the site quality in relation to the species of interest. Examples of such systems are that by Cajander (1926), Ure (1950), Daubenmire and Daubenmire, (1968) and Hodgkins (1961, 1970). Application of this approach is constrained by a number of factors. Deeper soil horizons, for example, may have little influence on understorey vegetation but still have great influence on the site for tree growing. Understorey vegetation is also influenced by such factors as wildfire, animal grazing and site preparation.

Topographic, climatic and edaphic factors can be used to estimate site quality. Data pertaining to such factors and forest stands are collected if possible in the field and site index regression equations then developed.

Theoretically, tree growth is controlled by the environmental factors such as soil moisture, nutrients (principally N, P and K), rainfall, temperature, aspect etc. These factors are used to predict site quality, and should, because of the logical approach, yield good results. However, very often the users of this sort of site index equation have to collect environmental values as inputs to the equation. This laborious, costly, and inconvenient (sometime impossible) need for certain data prevents its widespread application in practice, even though research on this topic is quite voluminous (e.g. Coile, 1952; Myers and Van Deusen, 1960; McGee, 1961; Lewis and Harding, 1963; Carmean, 1970; Steinbrenner, 1975; Alban, 1976; Clutter, et al., 1983).

All three above approaches evaluate site quality indirectly. Site quality can also be evaluated directly through use of historical yield records. But most forests lack such records, and even if available, changes in species selection, establishment density, silvicultural practices, rotation age, and genetic improvements etc. make this approach inapplicable in most cases at present.

For stands in which factors that influence volume production are strictly controlled, stand volume is an excellent indicator of site quality (Clutter et al., 1983; Lewis et al., 1976). To control those factors, however, the costs involved would be very high.

By far the most common approach is to estimate site quality from stand height, a method suggested long ago by

Roth (1916). It is based on the fact that volume production potential is positively related to height growth; in even aged stands the mean top height is little affected by density and intermediate cutting, except through thinning from above (Spurr, 1952). Thus mean top height could be a good indicator of site quality.

This approach involves the development of a set of site index equations, simply a family of curves of mean top height at a specified base age (see for example Bennett et al., 1959; Zahner, 1962; Coile and Schumacher, 1964; King, 1966; Brickell, 1968; Lundgren and Dolid, 1970; Beck, 1971; Carmean, 1972; Graney and Burkhart, 1973; Bailey and Clutter, 1974; Trousdell et al., 1974; Burkhart and Tennent, 1977; Newberry and Pienaar, 1978; Clutter and Jones, 1980; Borders, et al., 1984; Harrison. et al., 1986; Bailey et al., 1989).

With this approach, there are distinct methods of generating the indices. Clutter et al., (1983) generalized them into three kinds: (a) guide curves; (b) difference equations; (c) parameter predictions.

Analysis of this Douglas fir data set showed that no single one of these methods is best for all situations. Rather, a good fit results from a good combination of methods and equations with a particular data set. In this instance, the difference equation method and a Chapman-Richards height equation gave the best overall predictions.

Site index equations are classified into three types: anamorphic, polymorphic-disjoint and polymorphicnondisjoint (Clutter et al., 1983; Borders, et al., 1984). The equations developed in this study belong to the anamorphic type and they are now described in the following sections.

4.4.2.2 Calibration of existing site index equations

Burkhart and Tennent developed a site index equation for Kaingaroa Douglas fir forest in 1977 using the parameter prediction method and the Chapman-Richards equation:

$$h_{100}=\frac{S[1-\exp(-0.00116195ST_2)]^{1.8835}}{[1-\exp(-0.033941405(40))]^{1.8835}} \qquad \ldots\ldots\ldots\ldots (4.4.2.1)$$

but the authors were unable to say whether or not this equation could be applicable outside Kaingaroa. Calibration of the applicability of Burkhart's equation was first carried out, therefore.

Mean top heights were estimated for each measurement in Kaingaroa forest, using Burkhart's equation and by the new one provided here. Residuals pertaining to these estimates were then plotted to detect possible bias. Figures (4.4.2.1) to (4.4.2.3) are the residual patterns of Burkhart's equation for the three site groups: Kaingaroa, Whakarewarewa and Karioi (where RESIDB and PREDB stand for residuals and predicted values from Burkhart's site index equation). These figures show that the existing site index equation gives a good estimation for Kaingaroa forest, but not so good for the others. This led to the attempt to develop new site index equations for all the forests.

RESIDUAL PLOT OF SITE INDEX EQUATION

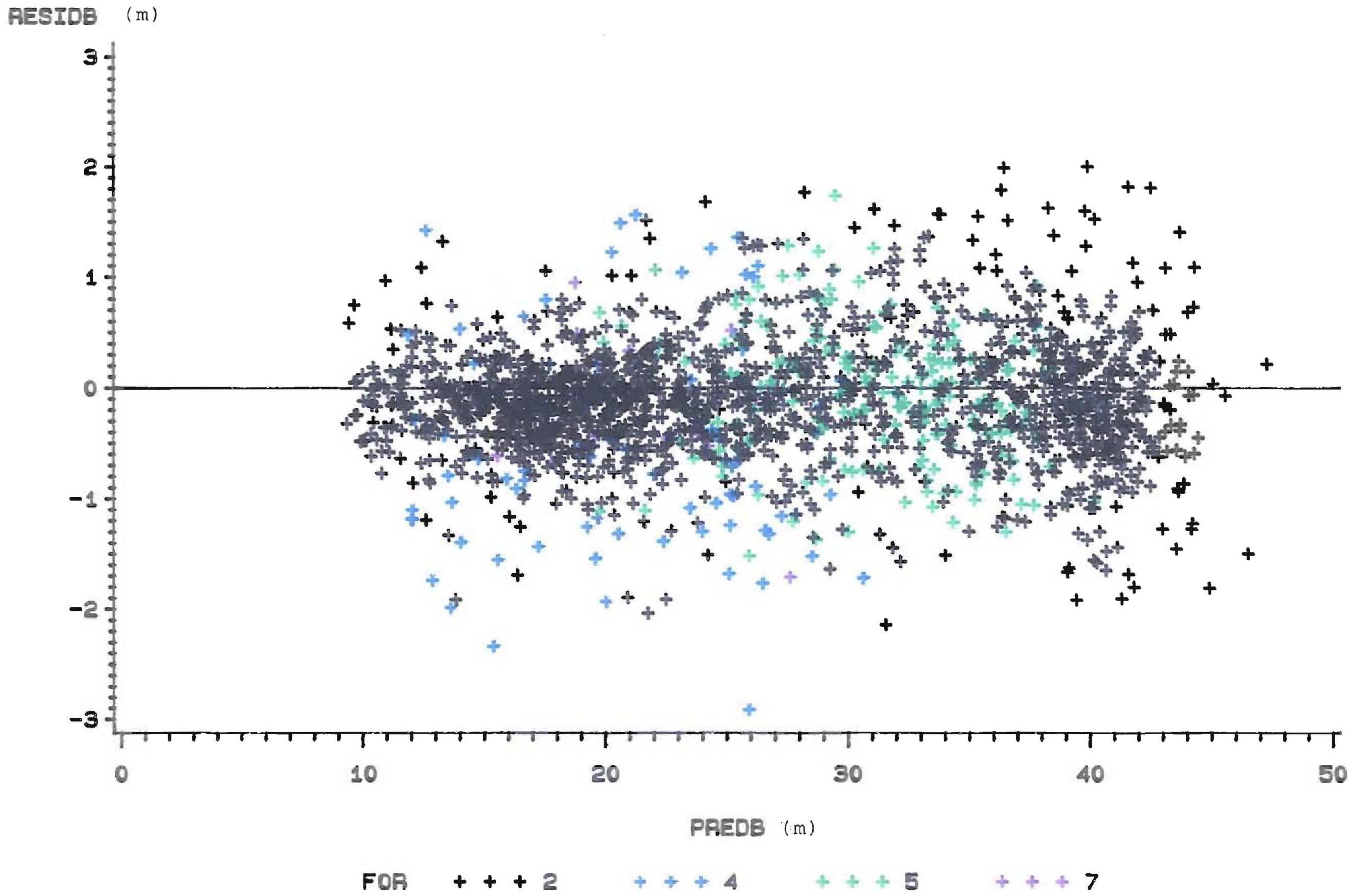


FIG (4.4.2.1) EXISTING EQUATION FOR KAINGAROA

RESIDUAL PLOT OF SITE INDEX EQUATION

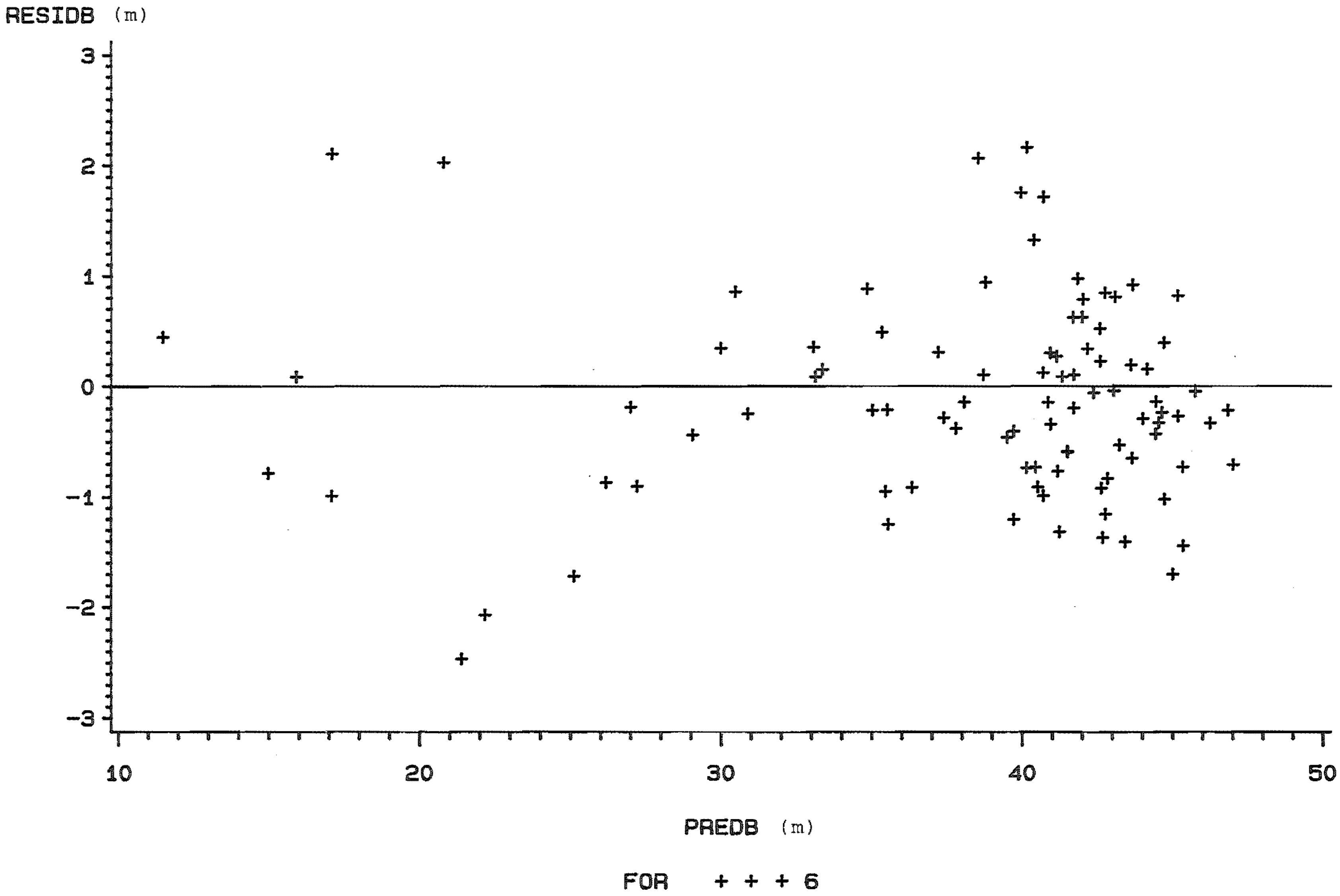


FIG (4.4.2.2) EXISTING EQUATION FOR WHAKAREWAREWA

RESIDUAL PLOT OF SITE INDEX EQUATION

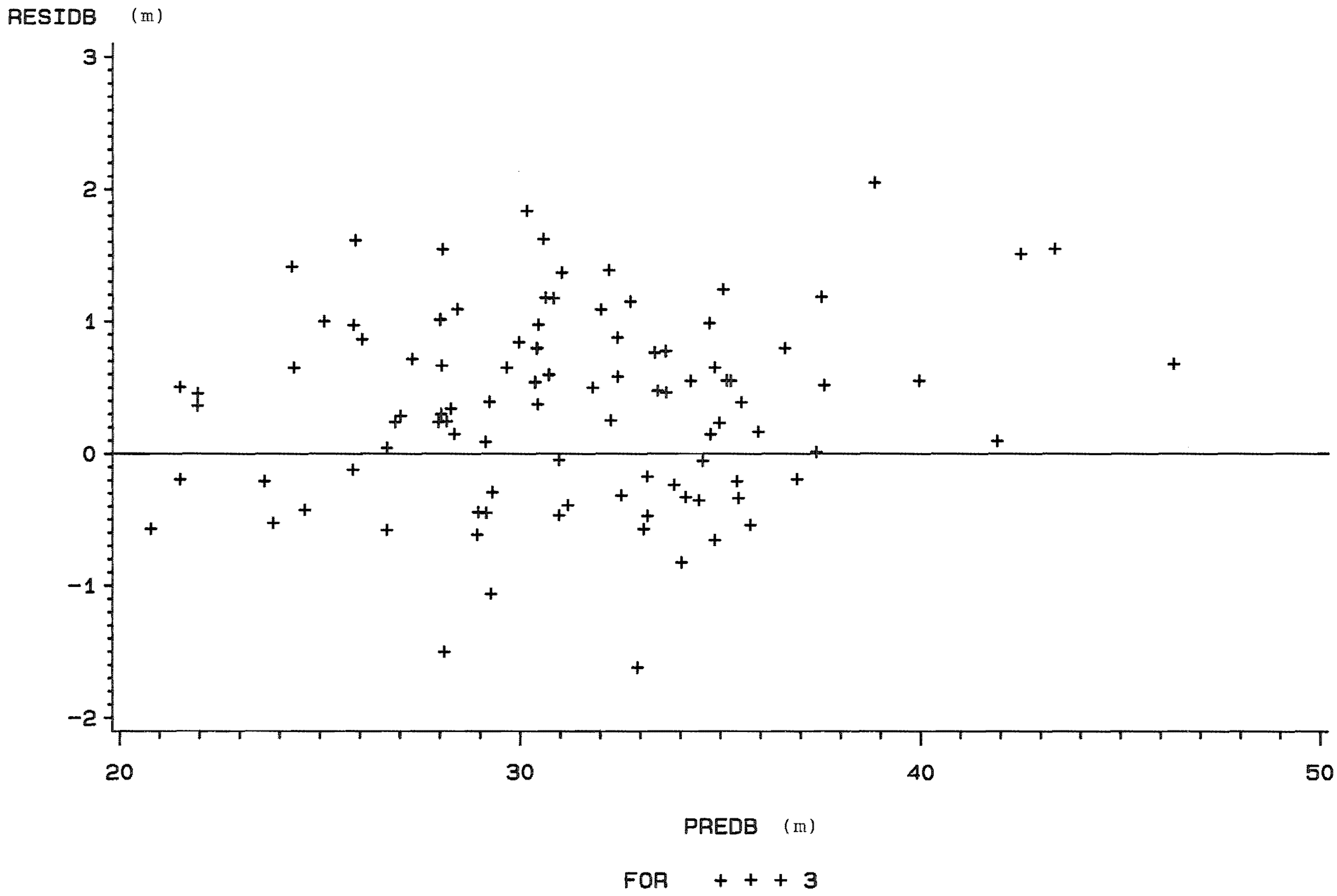


FIG (4.4.2.3) EXISTING EQUATION FOR KARIOI

4.4.2.3 Rationale of equations and methods

To ensure that the most appropriate method was used to fit the equations, several forms of equations have been fitted to the data using different methods. Some equations gave good fit with one method, some with another. The Schumacher equation,

$$h_{100,2}=\exp(\alpha+\frac{\beta}{T_2}) \quad \text{.............................} (4.4.2.2)$$

for instance, resulted in a better fit for the guide curve case, while the Chapman-Richards equation,

$$h_{100,2}=h_{100,1}[\frac{1-\exp(-\alpha T_2)}{1-\exp(-\alpha T_1)}]^{\beta} \quad \text{......................} (4.4.2.3)$$

gave the best fit when using the difference equation method.

where

$h_{100,1}$; $h_{100,2}$ = mean top height at T_1 and T_2;

T_1, T_2 = initial and remeasurement ages;

exp = exponent of natural logarithm;

α, β = coefficients to be estimated from the data.

Apparently, there is no particular method that is best for all situations; rather, the goodness of fit of a model depends on the appropriate combination of fitting methods, equations and nature of the data in specific circumstances.

In this study, the difference equation form of the Chapman-Richards function was employed as the final site index equation because this combination produced the best fit among several others tried.

4.4.2.4 Goodness of fit of the site index equation

Some previous research showed that mean top height of Douglas fir stands was not affected by the disease Phaeocryptopus gaeumannii (Manley 1985). For this large data set, plotting mean top height against age, as explained earlier, resulted in the same conclusion (Fig. 3.2.2.1). The mean top height equations were thus fitted to data without considering presence or absence of disease.

The final mean top height equation form is

$$h_{100,2}=h_{100,1}\left[\frac{1-\exp(-\alpha T_2)}{1-\exp(-\alpha T_1)}\right]^{\beta}\ldots\ldots\ldots\ldots\ldots\ldots(4.4.2.4)$$

which is given previously as equation (4.4.2.3). This equation was fitted to each of the three previously defined groups. Table (4.4.2.1) contains the coefficients and associated statistics of the equations. Fig (4.4.2.4) to (4.4.2.6) are the residual patterns for the new equations (where, RESIDN and PREDN stand for residual and predicted value of the new site index equation). They indicate that the equation fitted for Kaingaroa gave a good estimation, but the estimations for the other two localities were not so good. Comparison of the residual patterns for the new site index equations [fig (4.4.2.4) to (4.4.2.6)] with that

RESIDUAL PLOT OF MEAN TOP HEIGHT

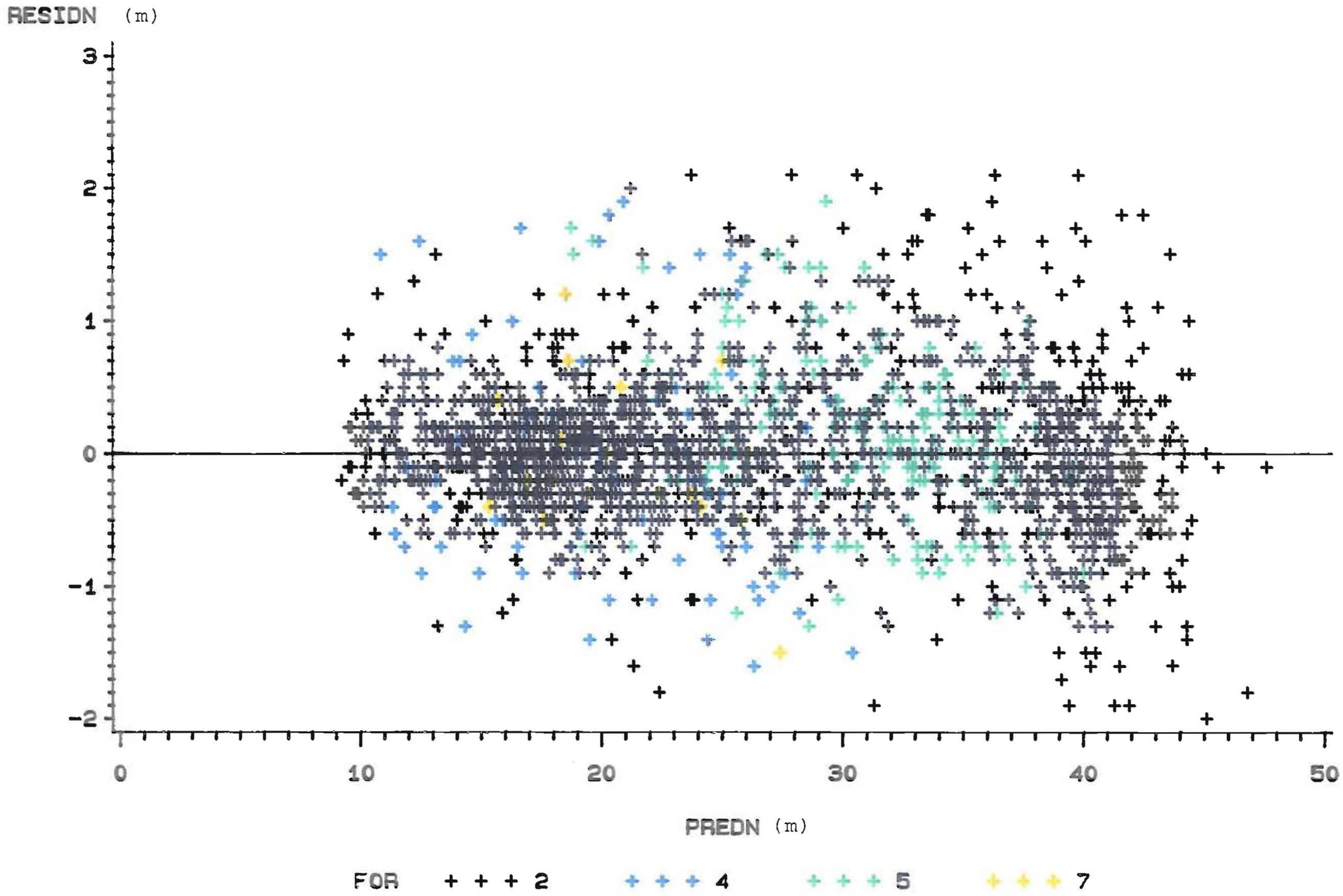


FIG (4.4.2.4) NEW SITE EQUATION FOR KAINGAROA

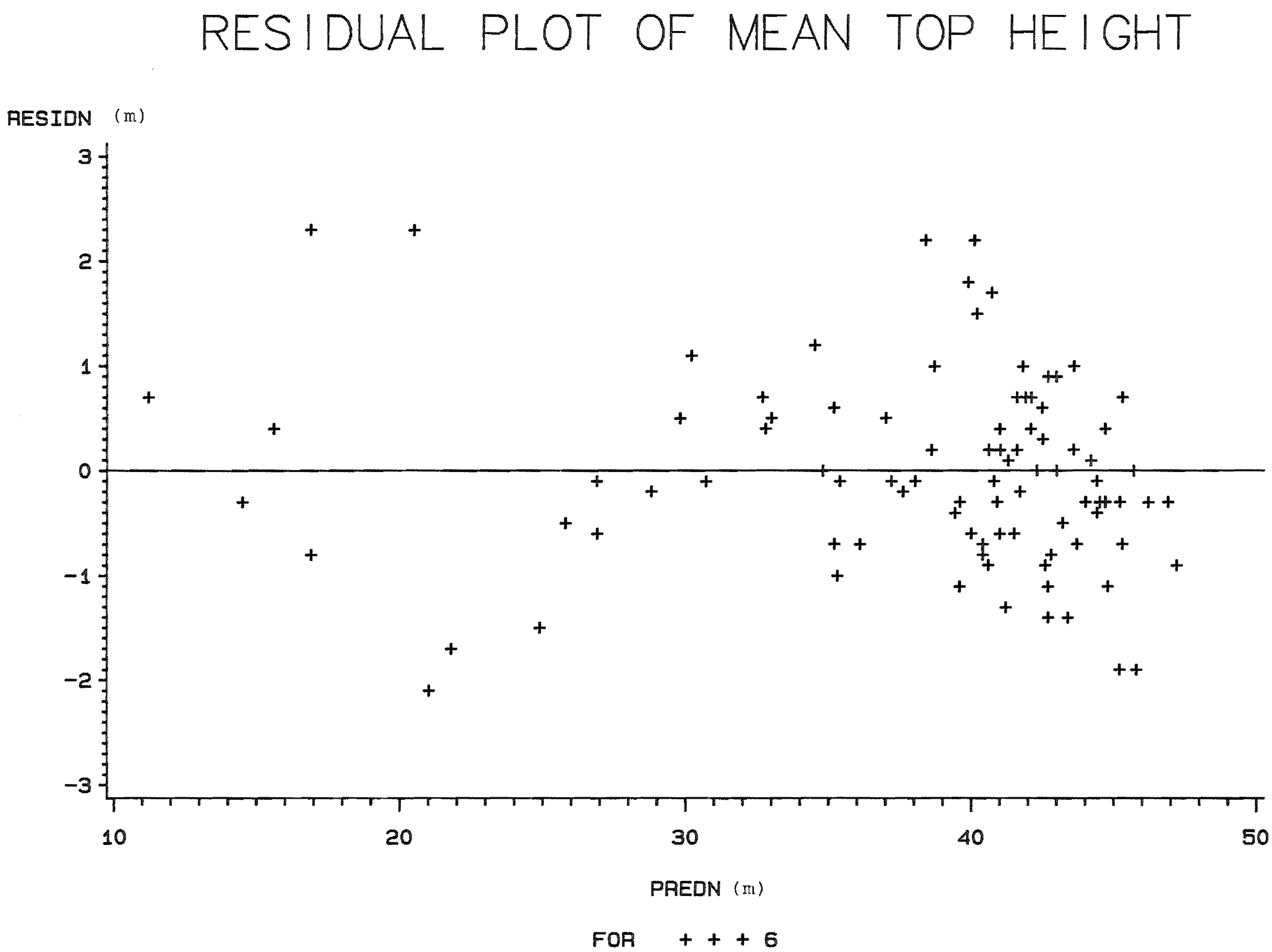


FIG (4.4.2.5) NEW SITE EQUATION FOR WHAKAREWAREWA

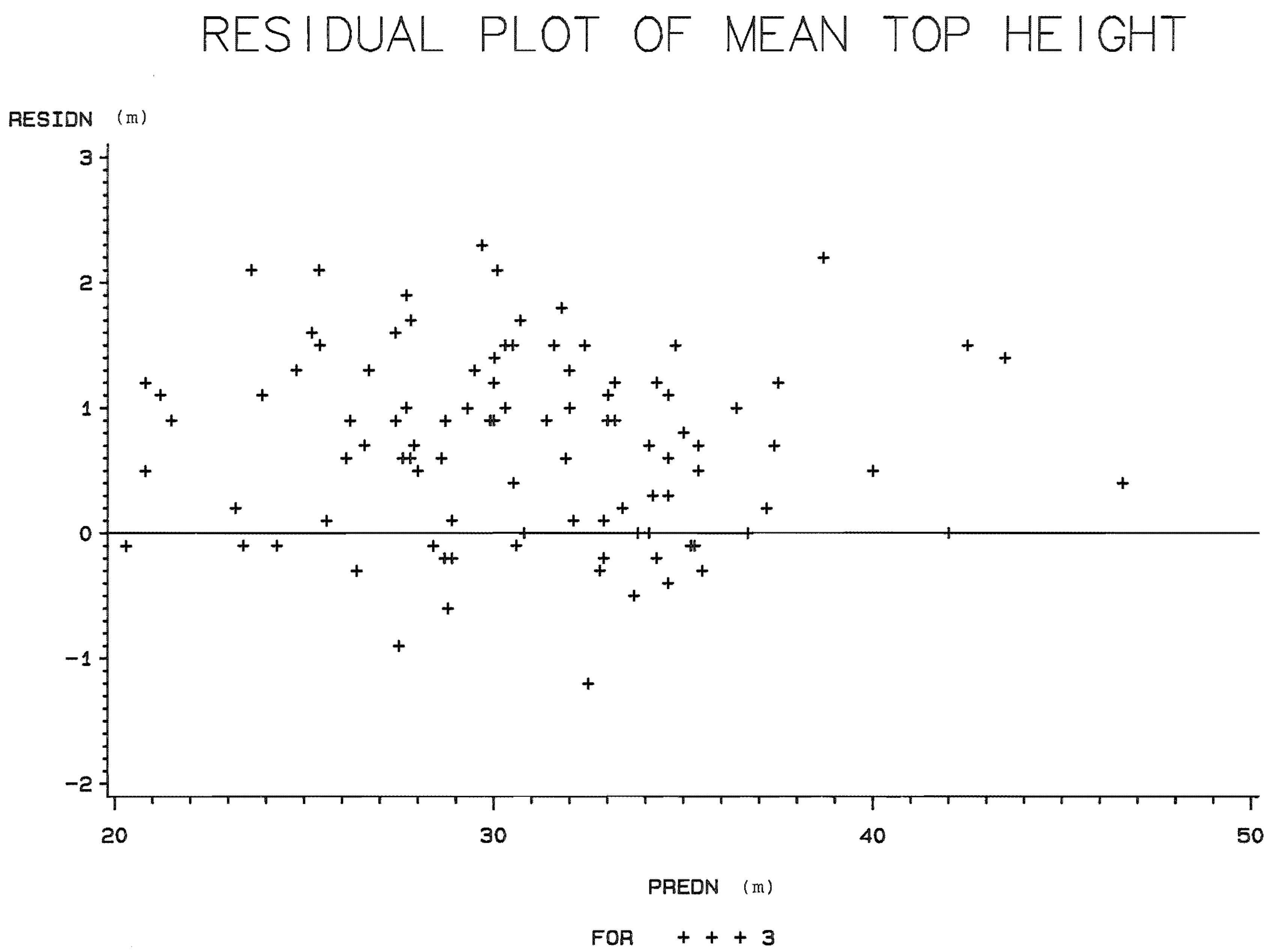


FIG (4.4.2.6) NEW SITE EQUATION FOR KARIOI

Table (4.4.2.1) Estimated statistics of new site index equations

Forest	Estimates of α	Estimates of β	STD error of α	STD error of β	n
KANG	0.03394	1.55301	0.00117	0.03315	1948
WAKA	0.03174	1.28186	0.00899	0.32411	96
KROI	0.01631	1.51261	0.00715	0.25956	104

of the existing equation [equation (4.4.2.1) and Fig (4.4.2.1) to (4.4.2.3)] shows that:

(1) both the new equation and Burkhart's equation give good estimation for Kaingaroa forest;

(2) the new equations and Burkhart's equation give almost identical estimations for all three groups: Kaingaroa, Whakarewarewa and Karioi;

(3) both the new equation and Burkhart's equation give identical estimation for Whakarewarewa and Karioi forests are indications: (1) both equations are capable of giving good prediction; (2) the bias shown on graphs for Whakarewarewa and Karioi may be due to the number of measurements for these two groups (96 and 104 respectively) being too small to reflect their intrinsic growth pattern as mentioned early in section 3.2.2.

When T_2 in equation (4.4.2.4) is set equal to T_0, the index age, $h_{100,2}$ is the site index by definition. i.e. with an index age 40 (Burkhart and Tennent, 1977; Mountfort, 1978)

$$S=h_{100,1}\left[\frac{1-\exp(-\alpha T_0)}{1-\exp(-\alpha T_1)}\right]^{\beta}\ldots\ldots\ldots\ldots\ldots\ldots(4.4.2.5)$$

Fig (4.4.2.7) is the site index curves drawn based on equations (4.4.2.5).

Although Burkhart's equation can give good estimations for Kaingaroa forest, it was decided to use the new site index equations in the models for the following reasons:

a. since more data are available for this project than when Burkhart and Tennent (1977) did their study, the new equation for the Kaingaroa group has wider application, covering also the forests in Pureora, Whirinaki and Waimihia, while the existing equation applies only to Kaingaroa;

b. the new equation gave a good estimation almost identical to that given by Burkhart's equation;

c. the new equation is more convenient to use as it can be explicitly solved for S, the site index.

d. unlike the existing one, the choice of the index age in the new site index equation has no affect on the shape of the site index curves.

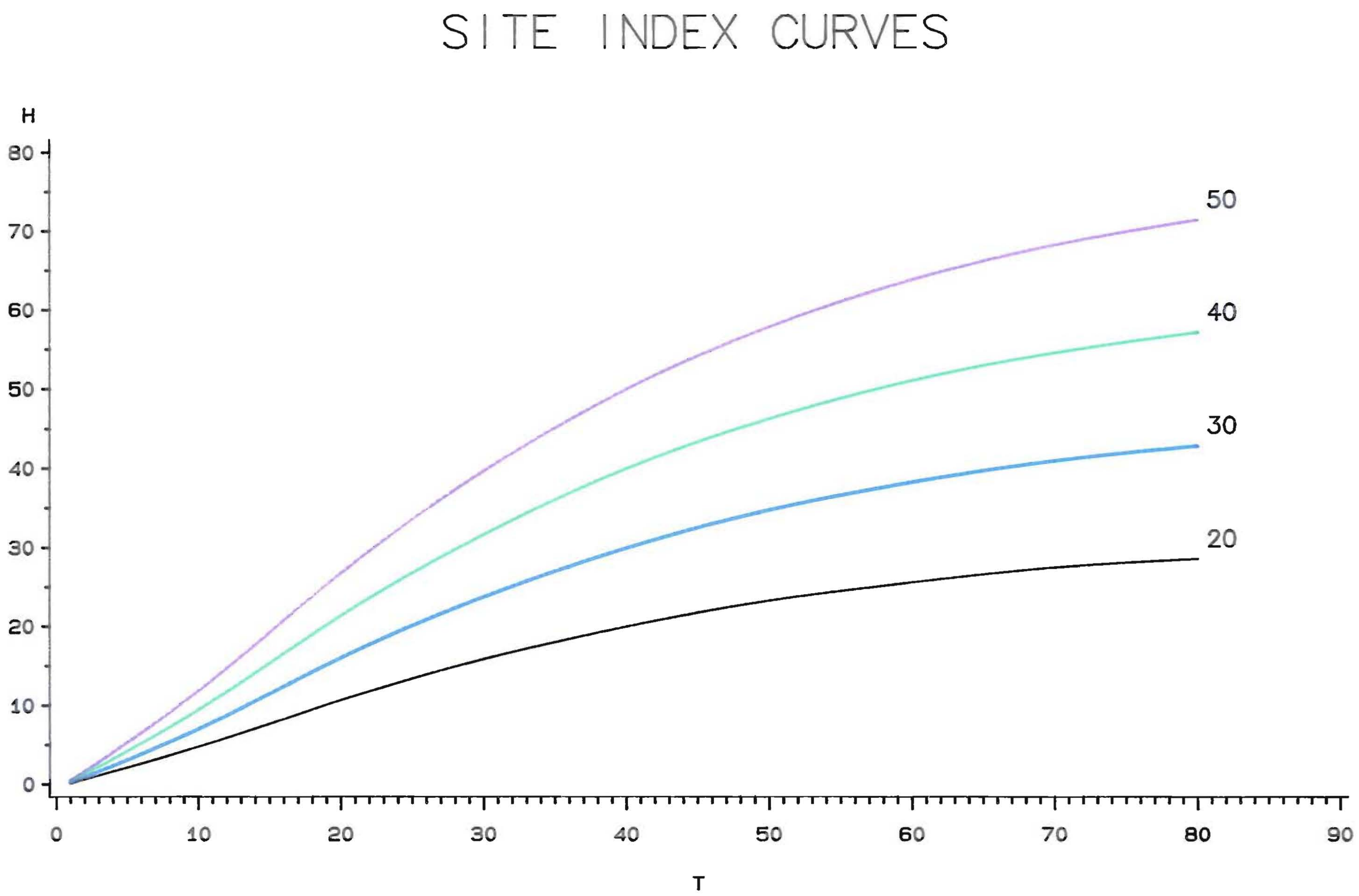


FIG (4.4.2.7): KAINGAROA REGION PLANTATIONS

4.4.3 Basal Area Projection Equations

4.4.3.1 Data for fitting basal area equations

Table (4.4.3.1) shows the numbers of observations available for fitting the basal area projection equations.

Table (4.4.3.1) Distribution of measurements by forest and thinnings

pre-1963						
Forest	thinning regime					
	none	1st	2nd	3rd	all 3	sum
Horohoro	0	0	0	0	0	0
Kaingaroa	41	58	26	3	87	128
Karioi	4	4	2	0	6	10
Pureora	0	0	0	0	0	0
Waimihia	7	6	3	0	9	16
Whakarewarewa	15	11	1	0	12	27
Whirinaki	0	0	0	0	0	0
sum	67	79	32	3	114	181

Table (4.3.2.1) (continued)

			post-1963			
Forest	thinning regimes					
	none	1st	2nd	3rd	all 3	sum
Horohoro	1	0	0	0	0	1
Kaingaroa	316	937	90	47	1074	1390
Karioi	78	0	0	2	2	80
Pureora	25	28	3	0	31	56
Waimihia	43	66	13	9	88	131
Whakarewarewa	12	42	10	2	54	66
Whirinaki	18	8	0	0	8	26
sum	493	1081	116	60	1257	1750

Because of the small numbers in the other two groups (also see section 3.2.2. and 4.4.2.4), again only the Kaingaroa group was considered here.

4.4.3.2 The use of the Hossfeld function as a basal area projection equation

Several equation forms were tried in fitting the basal area projection equations. Both the Schumacher equation

(2.1.6) and the Hossfeld equation (4.4.3.3) resulted in good fits. The Hossfeld function was finally chosen as the best fitting form for the basal area projection equation because it produced not only a slightly better fit but also has more desirable properties than the Schumacher equation (Woollons, Whyte and Liu Xu, 1990). The Hossfeld function is given by

$$Y=\frac{\alpha T\gamma}{\alpha\beta+T\gamma} \qquad (4.4.3.1)$$

Where Y = yield at time T and the rest of the symbols are parameters to be estimated. Differentiating (4.4.3.1) with respect to T gives the growth equation:

$$\frac{dY}{dT}=\frac{\alpha\beta\gamma Y}{T(\alpha\beta+T\gamma)} \qquad (4.4.3.2)$$

Difference equation forms of (4.4.3.1) are, with a thinning index introduced:

polymorphic,

$$Y_2=\frac{1}{(\frac{1}{Y_1})(\frac{T_1}{T_2})^{\gamma}+(\frac{1}{\alpha})(1-(\frac{T_1}{T_2})^{\gamma})+X(\frac{1}{\beta})(1-(\frac{T_1}{T_2})^{\gamma})} \qquad (4.4.3.3)$$

anamorphic,

$$Y_2=\frac{1}{(\frac{1}{Y_1})(\frac{1}{T_2^{\gamma}}-\frac{1}{T_1^{\gamma}})+\beta X(\frac{1}{T_2^{\gamma}}-\frac{1}{T_1^{\gamma}})} \qquad (4.4.3.4)$$

The Hossfeld equation has these properties:

(a) It is a sigmoid growth curve, with an upper asymptote, α,

and an inflexion point

$$Y = \lim \frac{\alpha T\gamma}{\alpha\beta + T\gamma} = \alpha \ldots\ldots (4.4.3.5)$$

$$Y_{inflex} = \frac{\alpha(\gamma-1)}{2\gamma} \quad at \quad T = [\frac{\alpha\beta(\alpha-1)}{\alpha+1}]^{\frac{1}{\gamma}} \ldots\ldots (4.4.3.6)$$

(b) as T2 approaches T1, Y_2 approaches Y_1;

(c) as T2 approaches ∞, Y_2 approaches an upper asymptote, α; (Schumacher, 1939; Clutter et al., 1983; Clutter and Sullivan, 1972; Knoebel et al., 1986; and others).

(d) Y = 0 when T = 0, In contrast, for the Schumacher equation, when T = 0, Y is not defined.

The yield equation, therefore, makes good biological as well as mathematical sense.

Although yield at age zero is not utilised in practice, (d) is a desirable property to have, just as the upper asymptote is seldom utilised, particularly for plantations. The role of the lower and upper asymptote is to force the yield function, when fitted to data, to fall within boundaries (i.e. lower and upper asymptotes) that should exist in reality for a biological population.

A thinning index was also introduced to the above equation, which simplified the model as a whole and increased the precision (see next section).

4.4.3.3 The fit of the Hossfeld

The Hossfeld function was fitted to the pre-diseased data of Kaingaroa group using the form of (4.4.3.3) with $1/\alpha$ and $1/\beta$ renamed α and β respectively. For the post-diseased group, the equation was fitted to the data with a modified form (4.4.3.7):

$$Y_2=\frac{1}{\left(\frac{1}{Y_1}\right)\left(\frac{T_1}{T_2}\right)^{\gamma+\frac{\delta}{T_2}}+\alpha\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma+\frac{\delta}{T_2}}\right)+X\beta\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma+\frac{\delta}{T_2}}\right)}\ . \quad (4.4.3.7)$$

where the thinning index, $X = 1- d_t/d_b$, d_t is dbhob of trees taking out in thinning and d_b is dbhob of the stand before thinning. Table (4.4.3.2) shows the estimates and standard errors for these equations. In the table, "pre-63" refers to undiseased stands and "post-63" for diseased

Table (4.4.3.2) Statistics of Hossfeld basal area projection equations (4.4.3.7)

Forest	Parameter Estimates and STD errors of				n
	α	β	Γ	δ	
post-63	0.00400 0.00044	0.00385 0.00035	0.85426 0.05203	22.4538 0.81050	1746 (854)
pre-63	0.00761 0.00041	0.00125 0.00032	2.59506 0.06754		157 (163)
post-63 (no X)	0.00837 0.00011		1.34375 0.04541	15.8956 0.84669	1746 (939)

stands, which "post-63 (no X)" is the same as "post-63" group but without using the thinning index, X, in equation (4.4.3.7). The residual sum of squares (figures in brackets) is reduced by 10% if the thinning index is used in the equation. Fig (4.4.3.1) to (4.4.3.4) are the residual plots and residual bar charts for diseased and healthy stands respectively.

In terms of residual patterns, the Hossfeld equation produced the best result compared with many others tried, but there are still discordant patterns on the plots of the residuals of the diseased group, most probably as a result of disease impact (see section 3.2.1). Previous studies (James and Bunn, 1978) showed that the disease had much less effect on the growth of young stands. Some results of this study show that the disease had little impact on the growth of stands before age 25 and growth decline thereafter. At about age 45 the decline reached about 30 % and this figure did not increase significantly afterwards (see section 7.4). Interestingly, the residual patterns for diseased stands (Fig 4.4.3.1) follow the same trend: the residuals are defined as actual values minus predicted. From age 5 to 25, because of less disease impact and more growth, the residuals are predominantly positive and lie on the upper side of reference line. Growth started to drop (fewer residuals are positive) at about age 25 and continue to drop until they reach the lowest point (more residuals are negative) at about age 40 and level off thereafter. It is expected that if disease information on the basis of plots or individual trees were available, the division of diseased and undiseased measurements could be made more precise and the fit of the basal area equation would be improved.

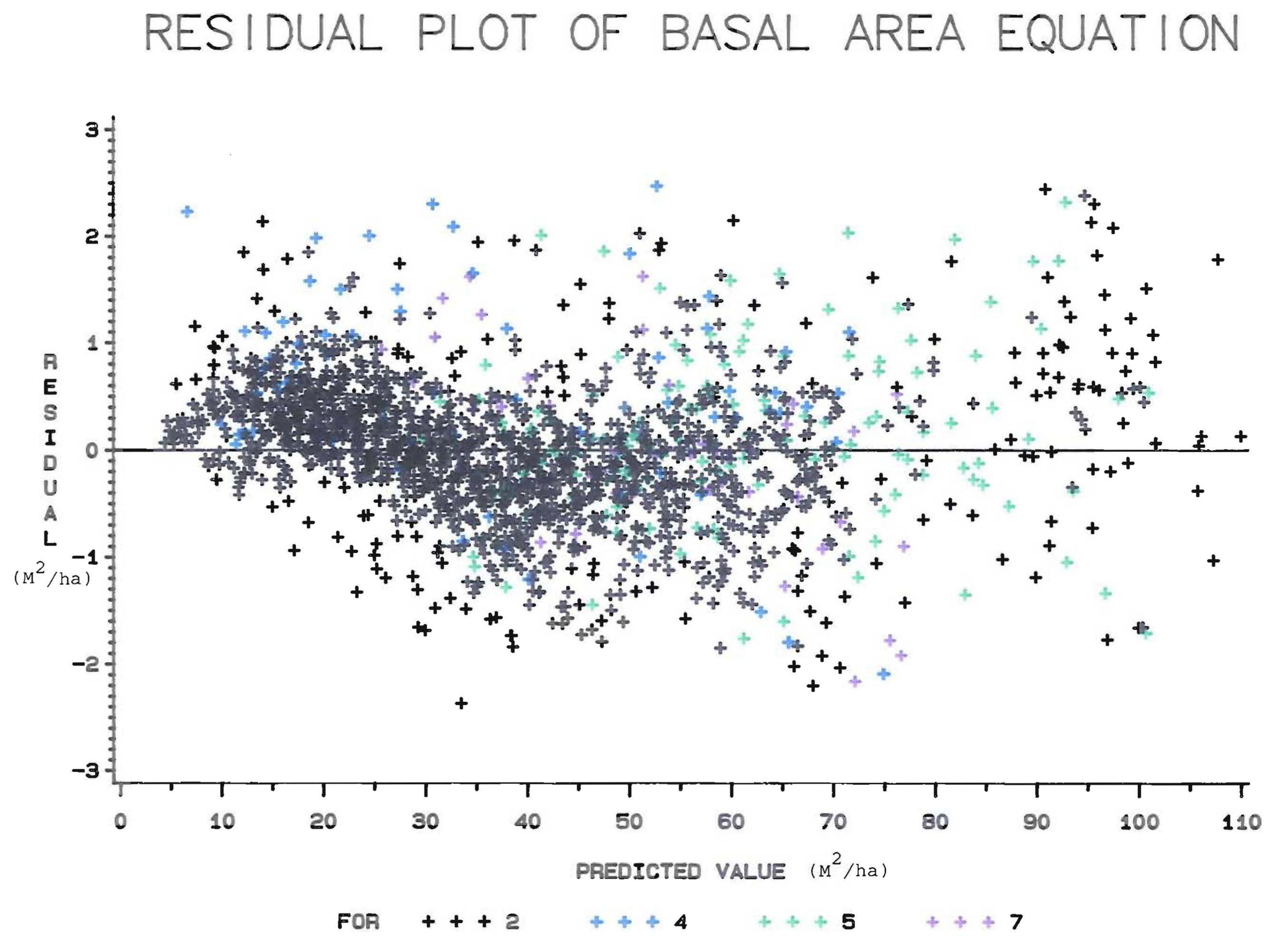


FIG (4.4.3.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF BASAL AREA EQUATION

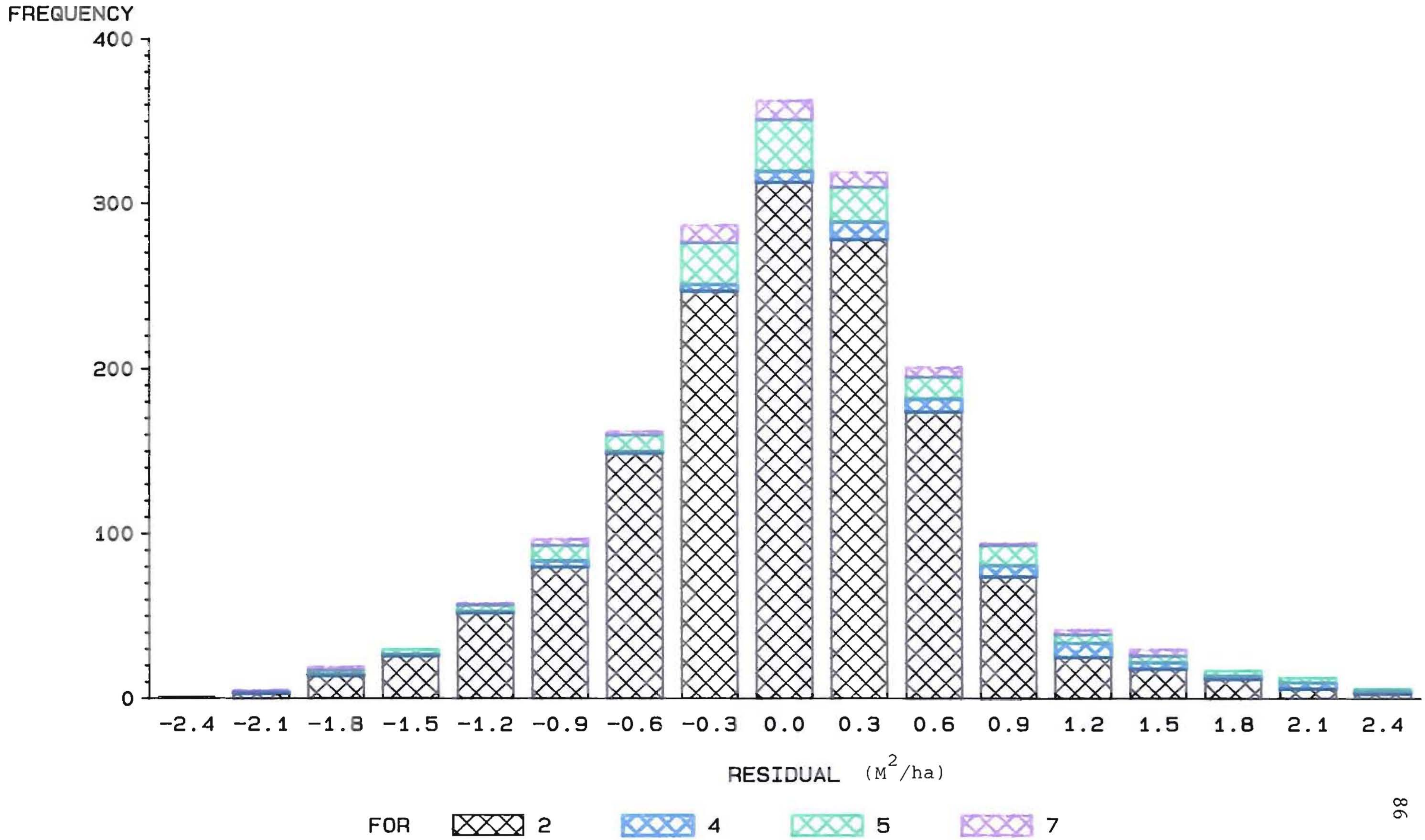


FIG (4.4.3.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF BASAL AREA EQUATION

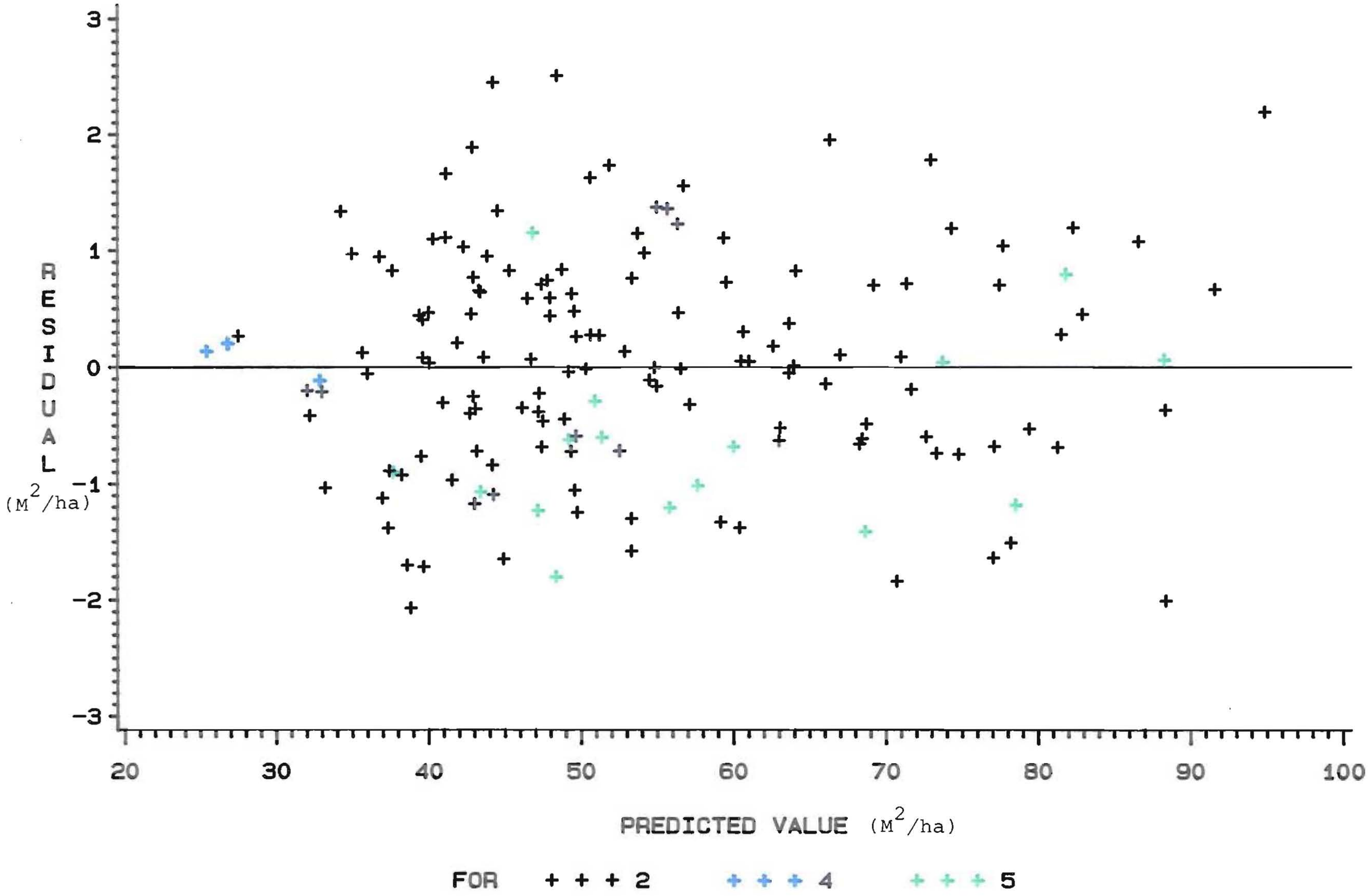


FIG (4.4.3.3): KAINGAROA UNDISEASED STANDS

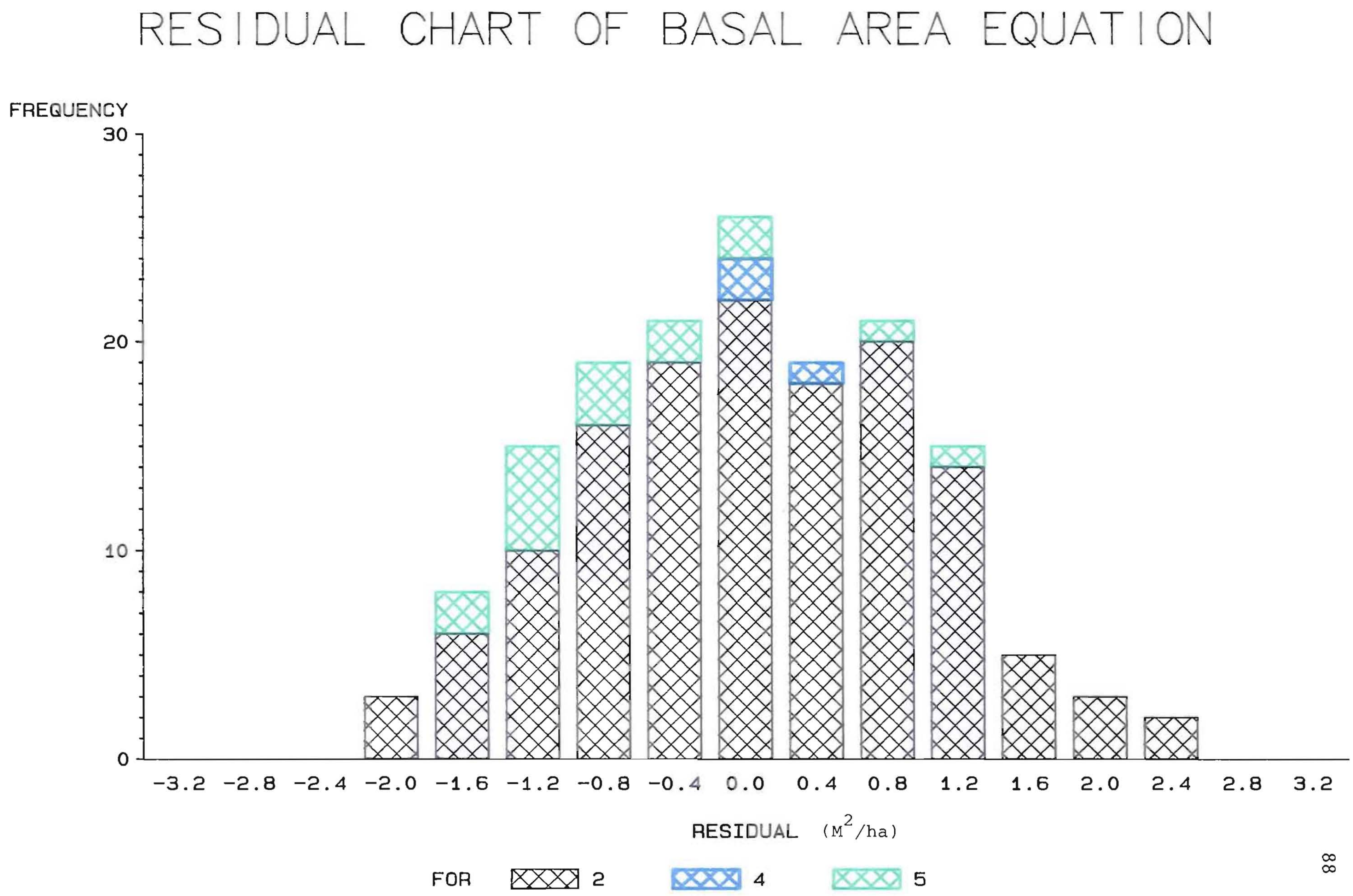


FIG (4.4.3.4): KAINGAROA UNDISEASED STANDS

4.4.3.4 Basal area/ha after thinning

Basal area/ha after thinning, if not specified, can be estimated from the following equation (Matney and Sullivan, 1982):

$$G_a = \alpha G_b^{\beta}(1-(1-N_a/N_b)^{\gamma})^{\delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.4.3.8)$$

Where

G_a = basal area/ha after thinning;

G_b = basal area/ha before thinning;

N_a = stems/ha after thinning;

N_b = stems/ha before thinning;

α, β, γ, δ = coefficients.

If desired, this equation can be rearranged to get the number of trees/ha after thinning.

Thinning data are summarised in table (3.1.1.2). The equation was fitted to the observations that involved a thinning between its starting (T_1) and ending (T_2) measurements. Table (4.4.3.3) presents the estimated coefficients and the standard errors of the estimates. Fig (4.4.3.5) and (4.4.3.6) display the residual pattern and residual bar chart of this equation. In order to make precise projections, it is recommended that users should supply their own inputs of basal area/ha and stems/ha before and after thinning because:

(1) basal area/ha after thinning estimated from stocking is an average value, while thinnings with different weights

is an average value, while thinnings with different weights and times could induce much variation;

(2) due to a small number of observations (123), the residual bar chart of this equation shows some bias and the parameter estimations are relatively poor. It is also recommended that users supply basal area/ha after thinning rather than stems/ha after thinning because the number of stems/ha is not closely associated with volume production, and is thus less important than basal area/ha.

(3) on average, stands covered by this data set were first thinned at about age 25. Second and third thinnings were carried out about 5 and 4 years later respectively (Table 3.1.1.2). This equation could produce even poorer result than the residual statistics indicates if thinning intervals are markedly different from the ones just described above.

Table (4.4.3.3) Statistics of equations for G/ha after thinning

Parameters	Estimates	STD error	n
α	1.575813766	0.15421165798	123
β	0.895126313	0.0252280853	
Γ	0.983393463	0.10315973274	
δ	0.646141197	0.03677562871	

RESIDUAL PLOT, G/HA AFTER THINNING EQUATION

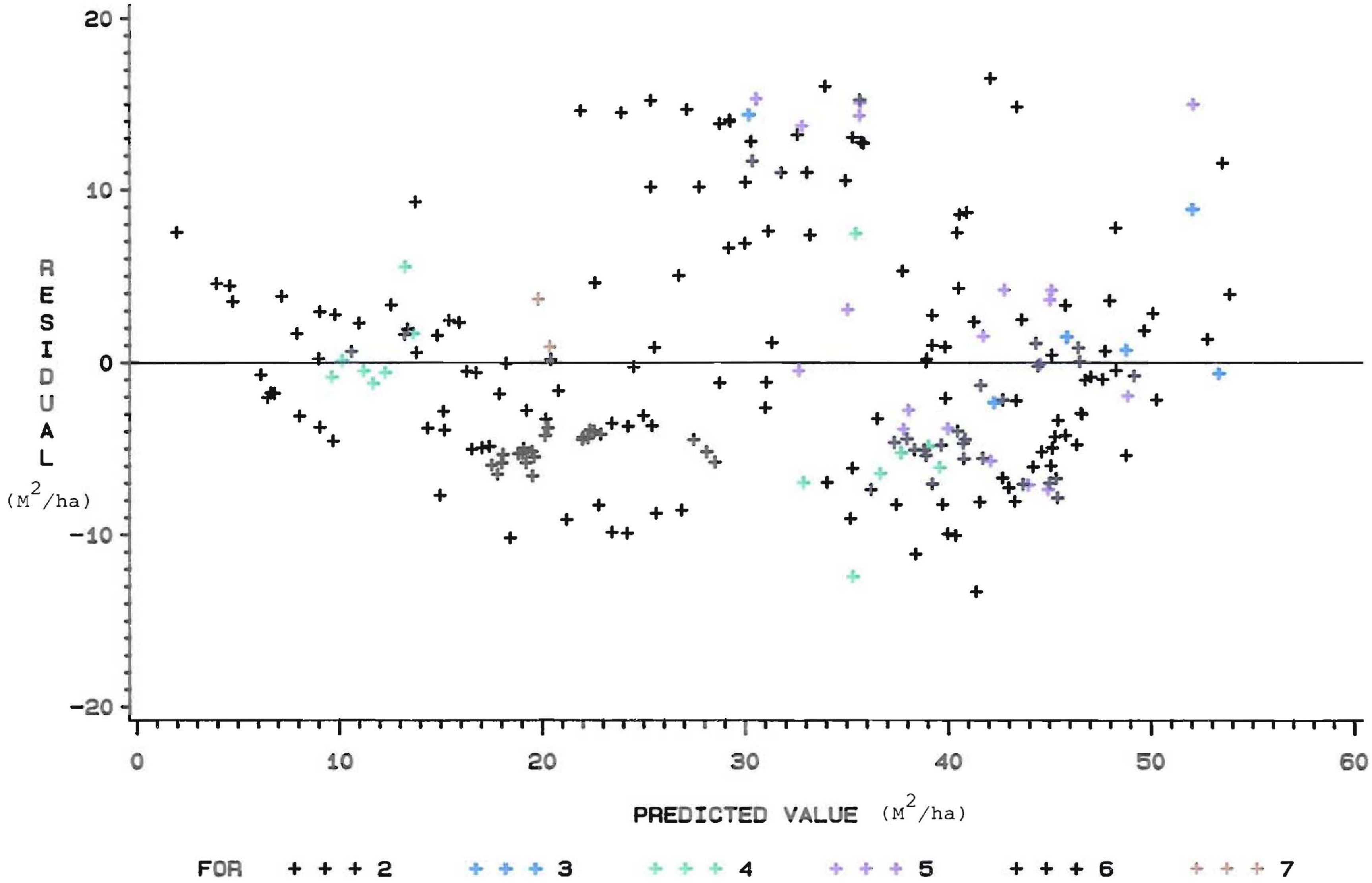


FIG (4.4.3.5): CENTRAL NORTH ISLAND PLANTATIONS

RESIDUAL CHART, G/HA AFTER THINNING EQUATION

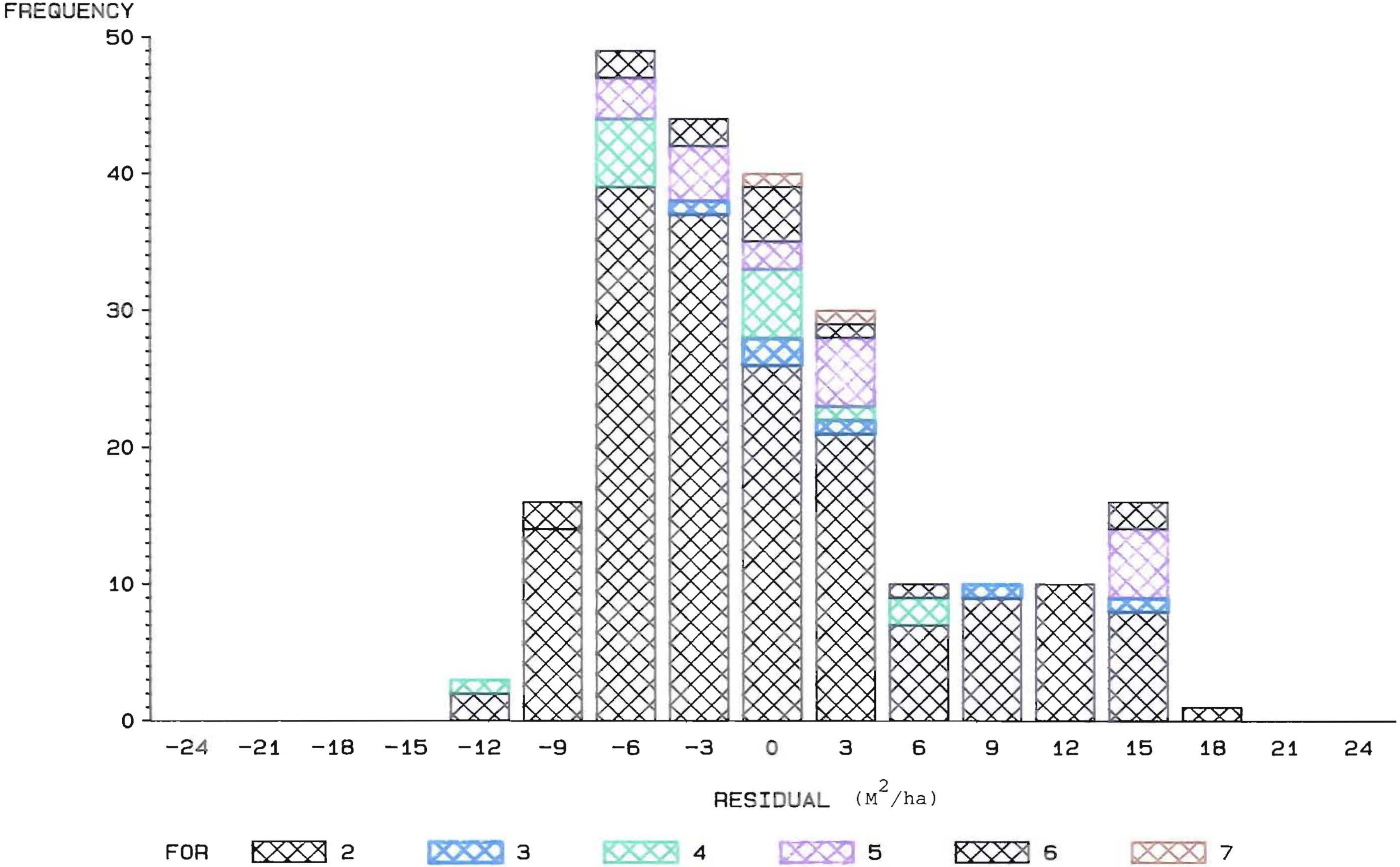


FIG (4.4.3.6): CENTRAL NORTH ISLAND PLANTATIONS

4.4.4 Volume Equations

Total stems volume/ha in the data set was calculated from a tree volume equation of the form (McEwen, 1976; Levack, 1986):

$$\ln(V) = b_1\ln(d) + b_2\ln(h^2/(h-1.4)) + b0 \quad .....(4.4.4.1)$$

where

$\ln(V)$ = natural logarithm of total volume inside bark;

d = diameter over bark at breast height (cm);

h = total tree height (m);

b_i = coefficients with values (Goulding and Gordon, pers. comm.) given in appendix 5.

The equation and coefficients applies to all Douglas fir plantations in New Zealand and high precision of estimation is unlikely when they are used just for the plantations in the Central North Island due to such differences as site quality (is lower in South Island) and disease effect (is much more severe in the North Island). This will have an effect on the stand equations developed in this study.

Stand volume equations were developed for diseased and undiseased stands based on the volume/ha data recorded in the permanent sample plots. Several equations, including one recommended by Sullivan and Clutter (1972) were tried during the course of the study. The following simple equation, however, produced the best fit of total stem volume / ha, V_t.

For diseased stands the equation was

$$V_t = \alpha + \beta h_{100} + \gamma G + \delta G h_{100} \quad ....(4.4.4.2)$$

For disease-free stands, the corresponding equation was:

$$V_t = \beta h_{100} + \gamma G + \delta h_{100} G \text{ .... } (4.4.4.2)$$

where

V_t = total volume inside bark (m^3/ha);

G = basal area (m^2/ha);

h_{100} = mean top height (m);

α, β, γ, δ = coefficients to be estimated from the data.

The equations were fitted to the data using weighted lease-squares regression with weight $k=(Gh_{100})^2$. Figure (4.4.4.1) to (4.4.4.4) are the residual patterns and bar charts for the volume equations for disease-free and diseased stands. Forecasts of projected volume/ha are derived from projected basal area/ha and mean top height. Table (4.4.4.1) shows the values of the coefficients for the volume equations fitted for the previously defined two groups.

Table (4.4.4.1) Coefficients of volume projection equations for diseased (post-63) and undiseased (pre-63) sets

Forest	Estimates and Standard error of				n
	α	β	γ	δ	
post-63	-1.6745 0.46228	0.27278 0.03728	0.72481 0.02297	0.33120 0.00113	1769
pre-63		0.17850 0.08928	0.64410 0.05345	0.33779 0.00183	163

RESIDUAL PLOT OF VOLUME EQUATION

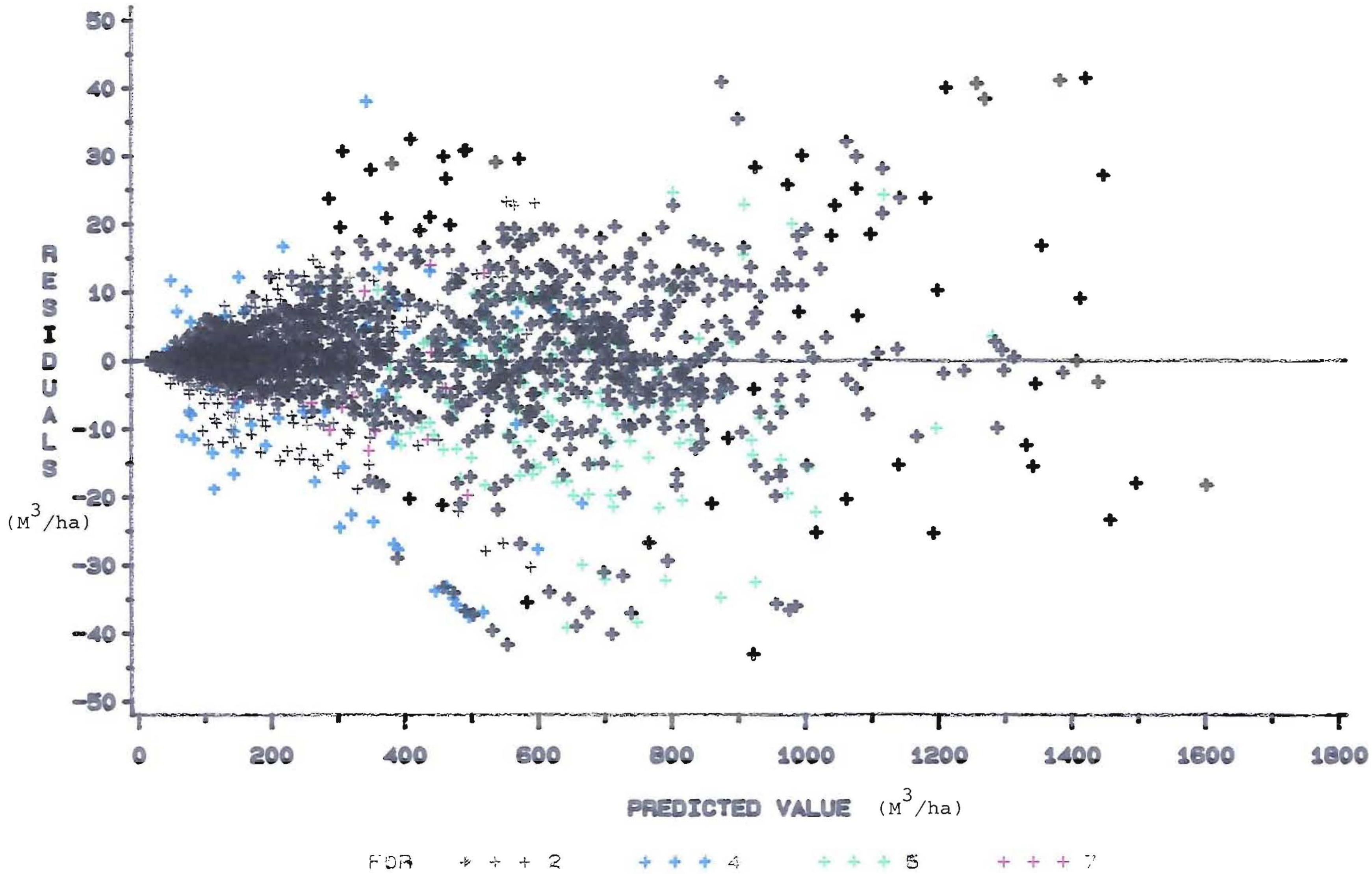


FIG (4.4.4.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF VOLUME EQUATION

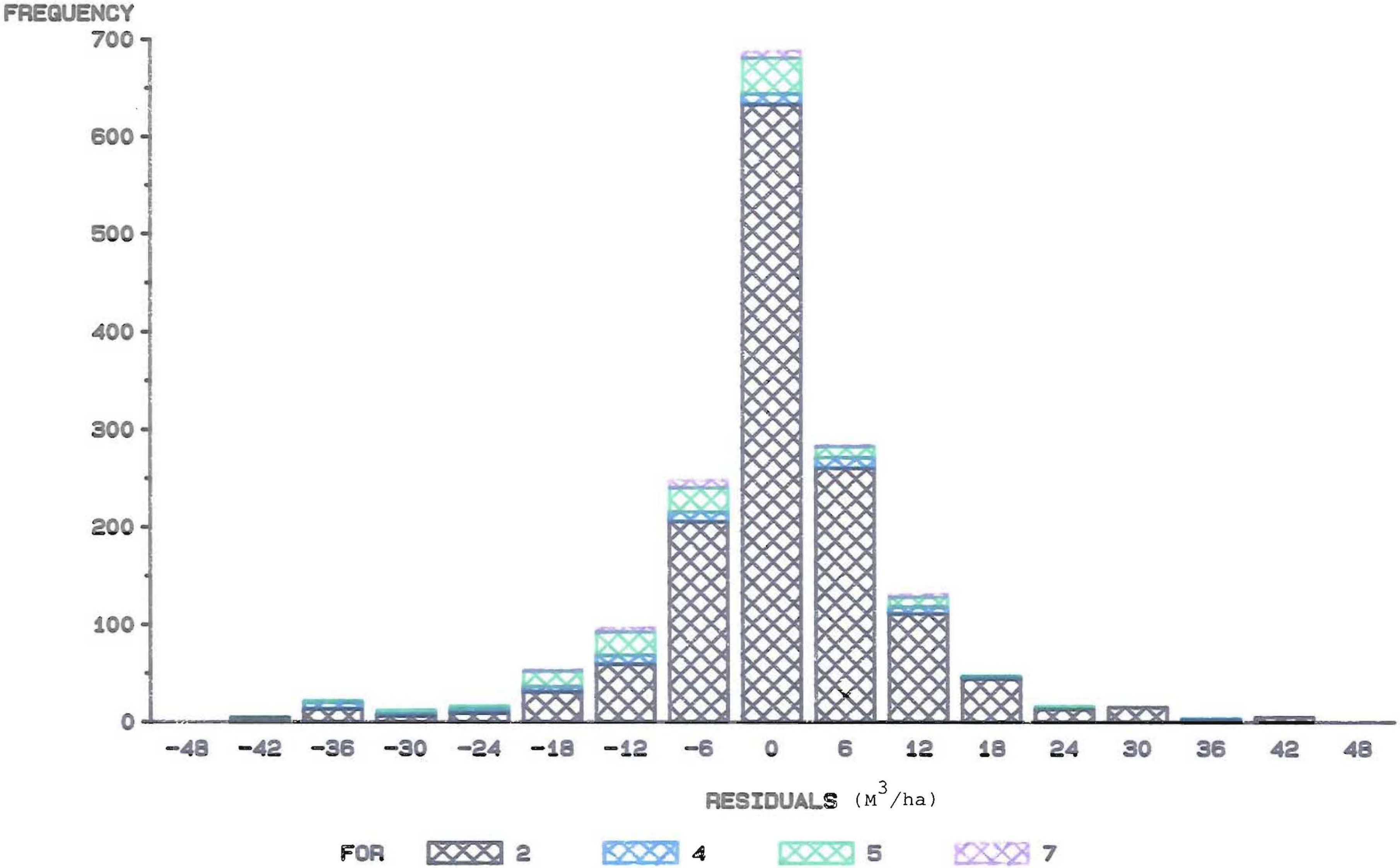


FIG (4.4.4.2):KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF VOLUME EQUATION

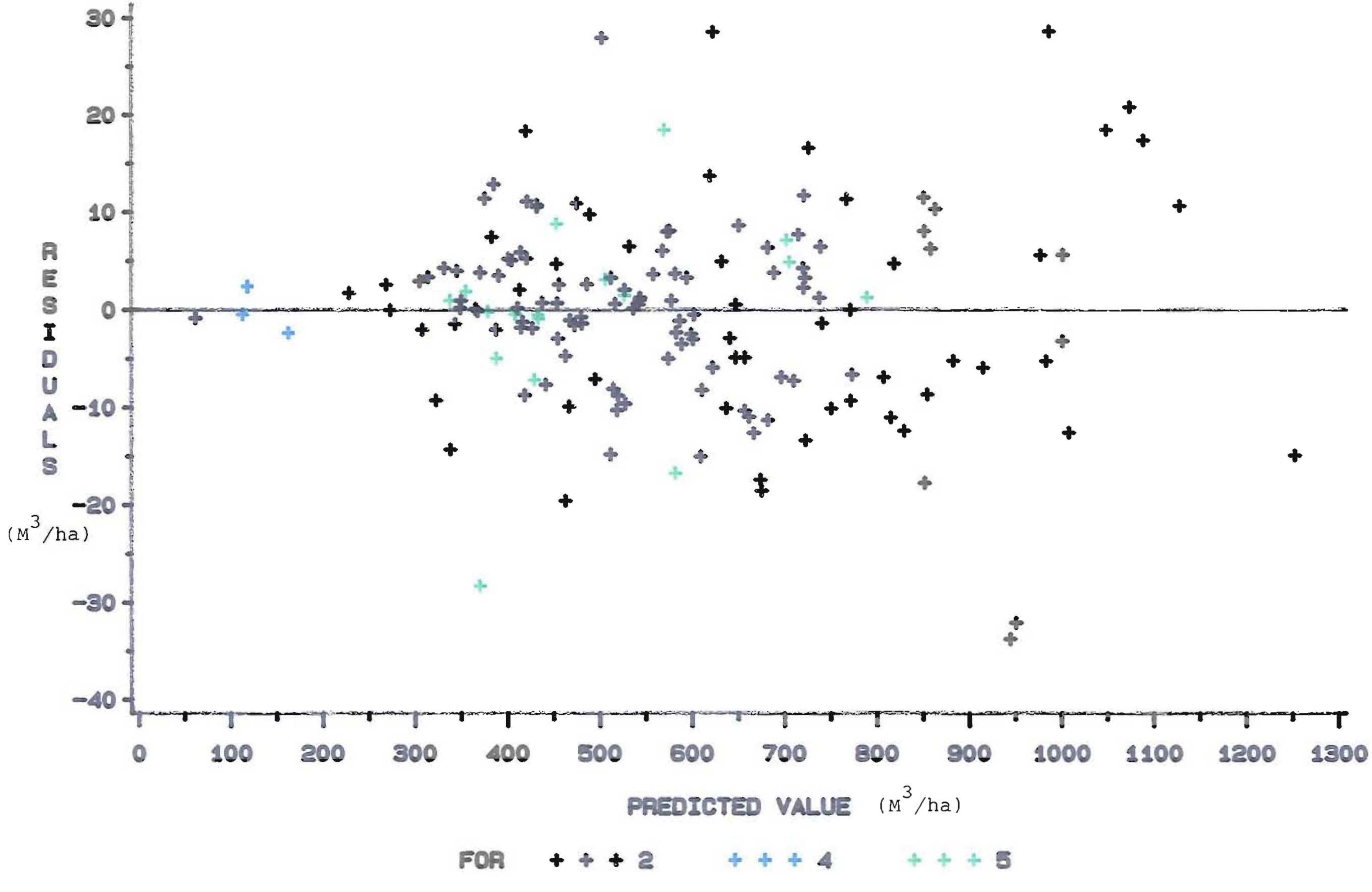


FIG (4.4.4.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF VOLUME EQUATION

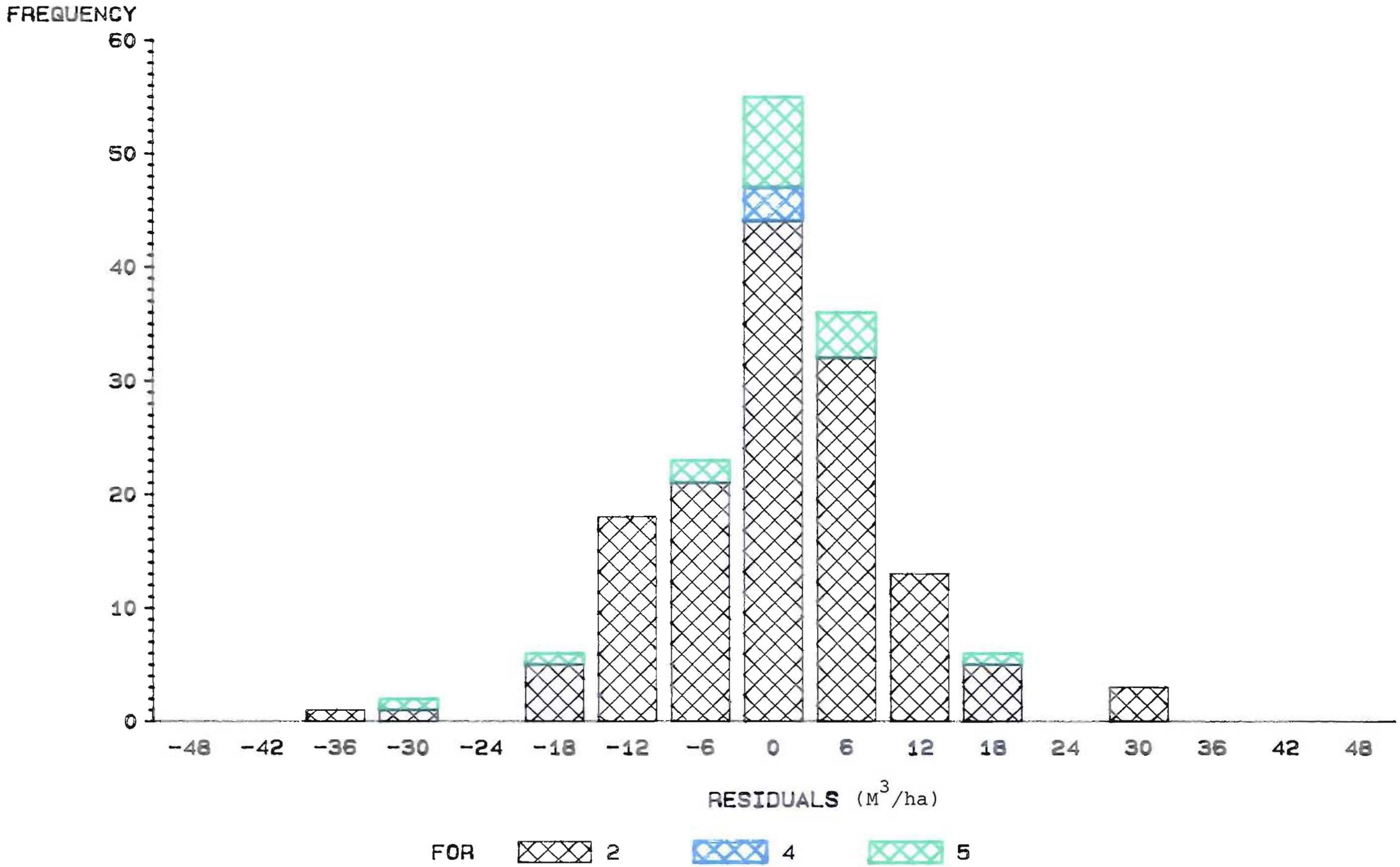


FIG (4.4.4.4):KAINGAROA UNDISEASED STANDS

4.4.5 Merchantable Volume Equations

The original data contained no information about merchantable volumes, which had, therefore, to be estimated from alignment charts constructed by Lewis (1954). These estimations could be far less reliable than those estimated from actual measured data.

The two data groups have the same form of merchantable volume equation:

$$V_m = \alpha V_t^{\beta} \exp\left[-\gamma\left(\frac{15}{d^{\delta}}\right)\right] \quad (4.4.5.1)$$

Where

V_m = merchantable volume inside bark (m^3/ha) to a 15 cm small end top diameter;

V_t = total volume inside bark (m^3/ha);

d_g = dbhob (cm);

α, β, γ, δ = coefficients estimated from the data.

Fitting of this equation showed that:

(1) it results in an excellent fit [Fig. 4.4.5.1 to 4.4.5.4];

(2) it always gives a logical estimation: e.g. the merchantable volume given by this equation is always reasonably lower than total volume.

Table (4.4.5.1) shows the coefficients for the merchantable volume equations for the two groups.

Table (4.4.5.1) Coefficients of merchantable volume projection equations

Forest	Estimates and STD error of				n
	α	β	γ	δ	
post-63	0.99352 0.00093	1.00008 0.00014	-427.89 3.97335	3.13603 0.00313	1653
pre-63	1.00071 0.00278	0.99975 0.00042	-313.30 17.5939	3.02983 0.01795	157

RESIDUAL PLOT OF MERCHANTABLE VOLUME EQUATION

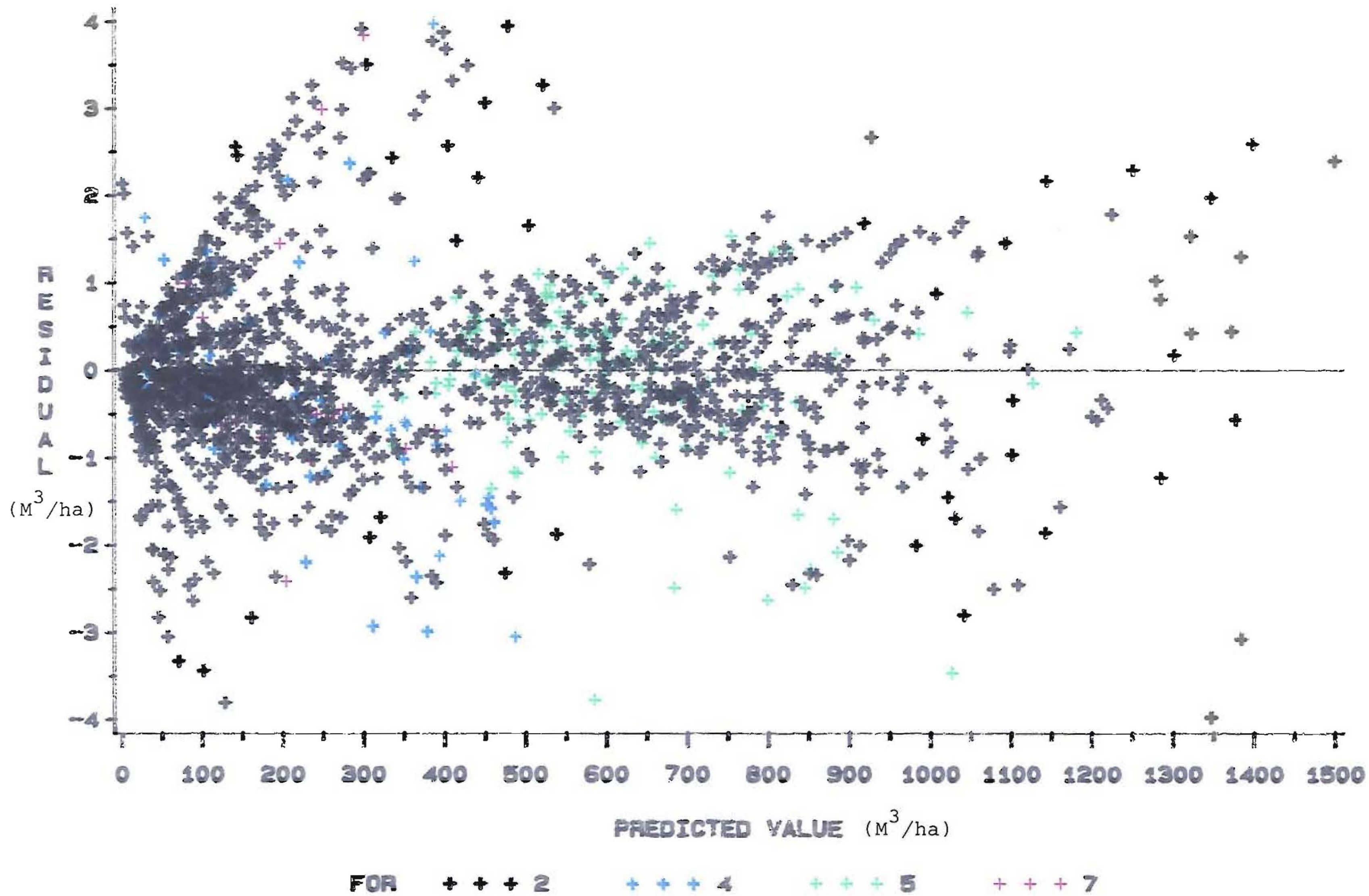


FIG (4.4.5.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF MERCHANTABLE VOLUME EQUATION

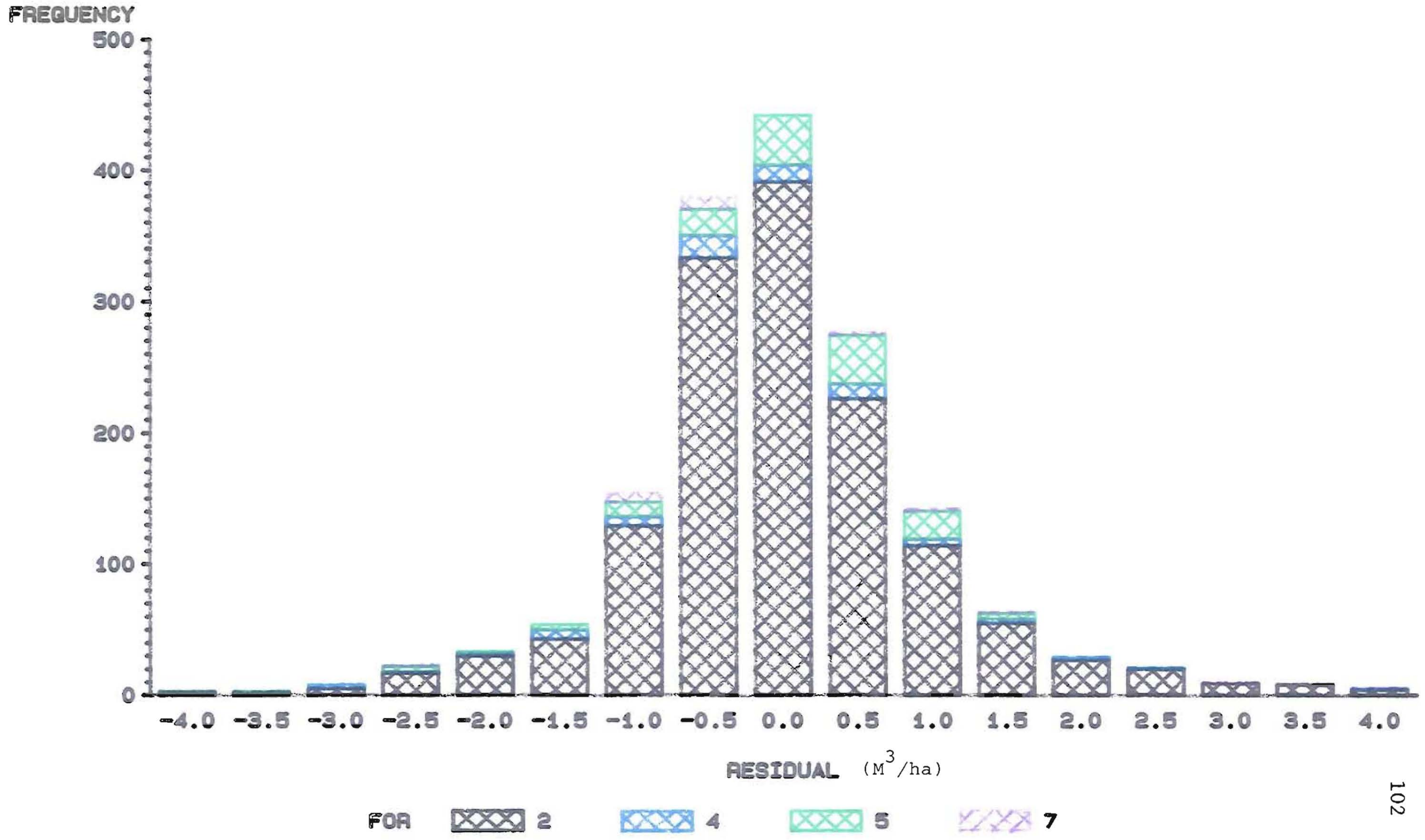


FIG (4.4.5.2): KAINGAROA DISEASED STANDS

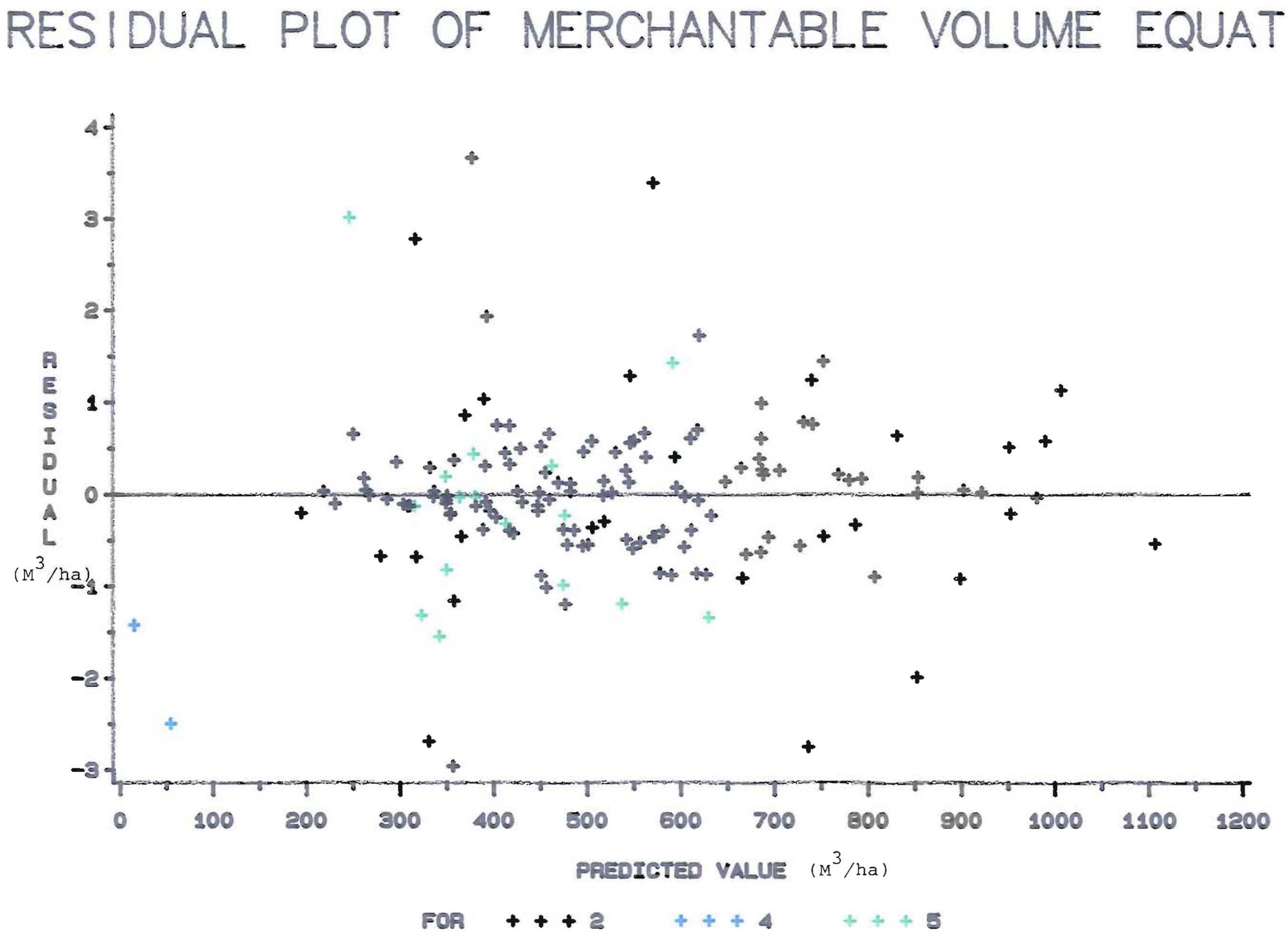


FIG (4.4.5.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF MERCHANTABLE VOLUME EQUATION

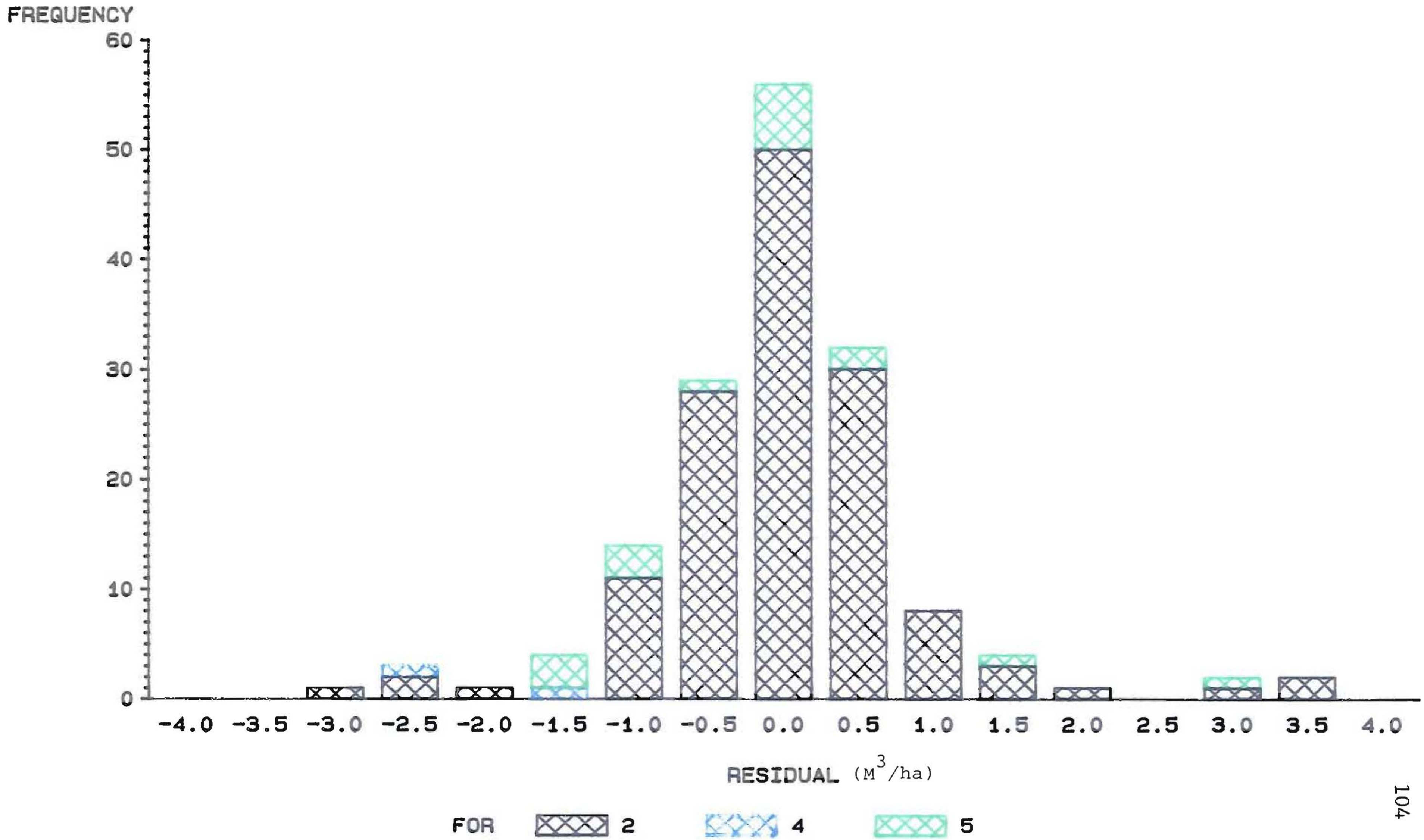


FIG (4.4.5.4): KAINGAROA UNDISEASED STANDS

4.4.6 Mortality Function

The mortality equation was formulated by first assuming a mortality rate (Clutter et al., 1983; Bailey et al., 1985) viz

$$\frac{dN/dT}{N}=\beta_0+\beta_1 G+\frac{\beta_2}{T}+\beta_3 S \ldots\ldots\ldots\ldots (4.4.6.1)$$

then integrating (4.4.6.1) which resulted in:

$$N_2=N_1\left(\frac{T_2}{T_1}\right)^{\beta_2} e^{(T_2-T_1)(\beta_0+\beta_1 G+\beta_3 S)} \ldots\ldots\ldots\ldots (4.4.6.2)$$

The thinning index, X, was introduced into (4.4.6.2) where

X = $1-d_t/d_b$;

d_t = mean diameter of thinned trees;

d_b = mean diameter of the stand after thinning;

so that the equation becomes

$$N_2=N_1\left(\frac{T_2}{T_1}\right)^{\beta_2} e^{(T_2-T_1)(\beta_0+\beta_2 G+\beta_3 S)X} \ldots\ldots\ldots\ldots (4.4.6.3)$$

(4.4.6.3) was further modified into

$$N_2=N_1\left(\frac{T_2}{T_1}\right)^{\beta_2 (T_2-T_1)(\beta_1 G+\beta_3 S+\beta_4 d_g)X} e^{(T_2-T_1)(\beta_1 G+\beta_3 S+\beta_4 d_g)X} \ldots\ldots (4.4.6.4)$$

where

G =1/(periodic basal area increment);

d_g = quadratic mean dbhob of the stand(cm).

The above procedure for deriving the mortality equation had been proposed by Clutter and others (1983) and applied by other researchers such as Bailey et al., (1985). But the equation formulated here is slightly different in that it contains the variables X and G, inclusion of which much improves the predictability. The equation reduced the residual sums of squares by 37 percent comparing to the equation used in DFCNIGM1.

The estimates for this equation are given in the following table:

Table (4.4.6.1) Estimated statistics for mortality equation

Parameters	Estimates	STD error	n
β_1	0.000236748	0.00003740667	790
β_2	-2.038555111	0.8614757985	
β_3	-0.000532254	0.0006971255	
β_4	-0.021408309	0.00213324567	

The residual patterns of the mortality equation are shown on the graphs (4.4.6.1) and (4.4.6.2).

RESIDUAL PLOT OF MORTALITY EQUATION

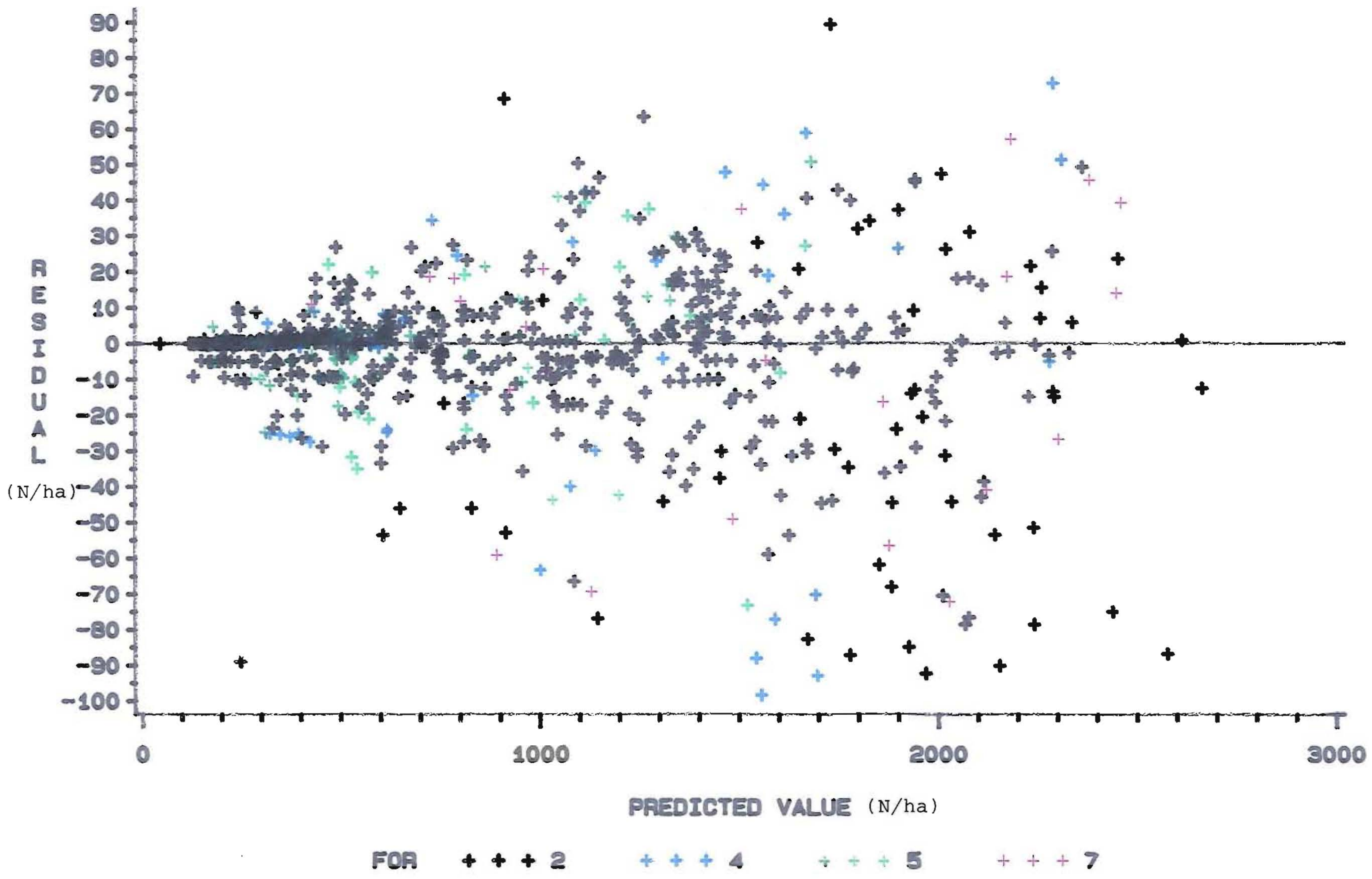


FIG (4.4.6.1): CENTRAL NORTH ISLAND PLANTATIONS

RESIDUAL CHART OF MORTALITY EQUATION

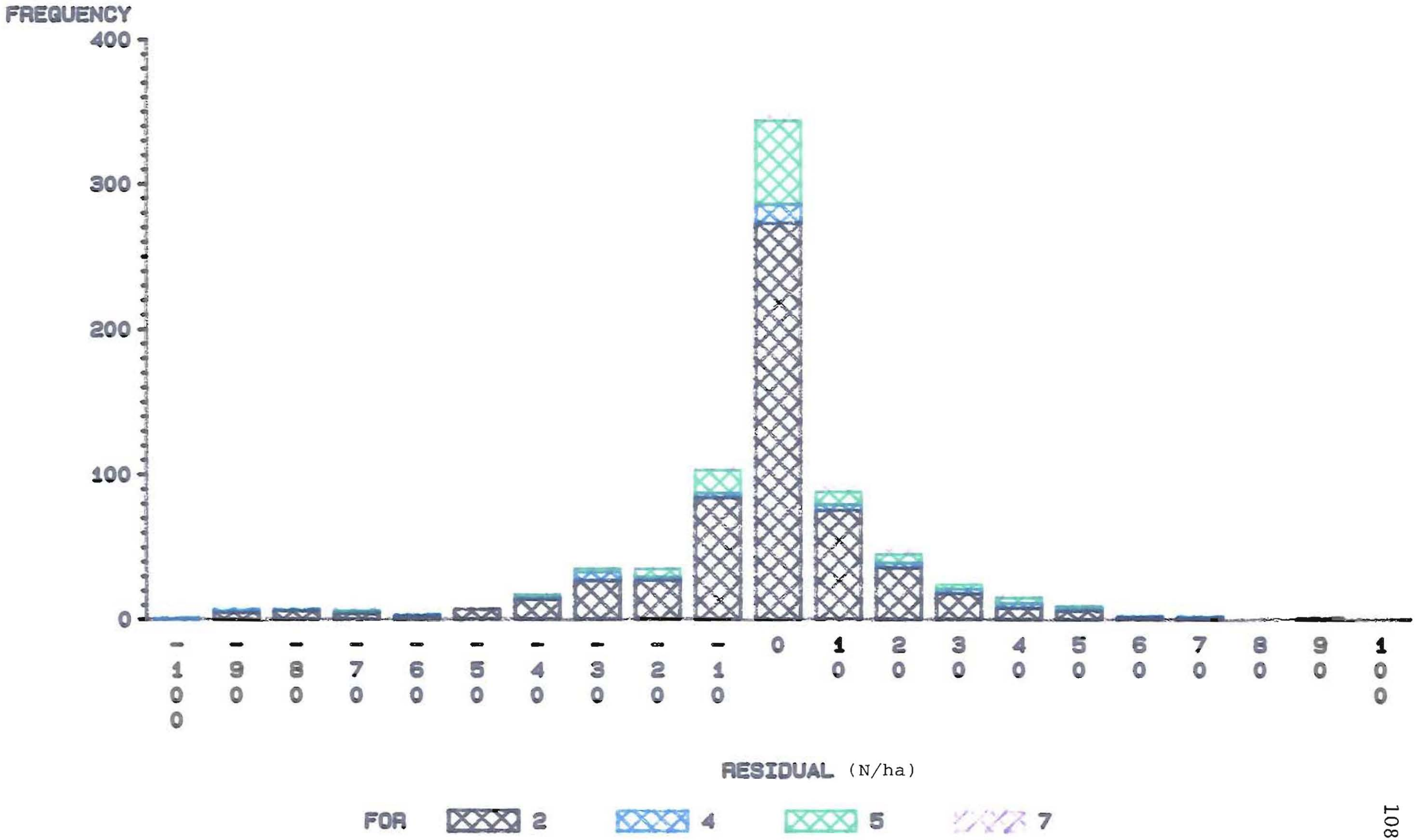


FIG (4.4.6.2): CENTRAL NORTH ISLAND PLANTATIONS

4.5 RELIABILITY OF THE MODEL COMPONENTS

With respect to the data used to fit the various components, 97% of the residuals of the site index equation lie within ± 1 m. On average, the predicted error lies within ± 0.42 m. For the Hossfeld basal area equation, all of the residuals lie within ± 2.4 m^2/ha. For estimating basal area after thinning with a stems/ha removal, 95 per cent of the residuals lie within ± 5 m^2/ha. User supplied inputs, therefore, are recommended or specification of basal area/ha removal is preferred when using the model to perform the thinning simulation. The total stem volume equation has nearly all its residuals within ± 40 m^3/ha for diseased stands and 30 m^3/ha for undiseased stands. The merchantable volume equation has most of its residuals within ± 4 m^3/ha (in addition to that from total volume equations) for both diseased and healthy stands. The revised mortality equation reduced the residual sum of squares by 37 per cent compared with that used in DFCNIGM1. In forecasting future stems/ha, 95 per cent of the residual lie within 70 stems/ha. Thus, the stand level components of DFCNIGM3 produce precise stand level yield forecasts for each of the types of Douglas fir stand in the Central North Island of New Zealand. This also ensures precise forecasts of the diameter distribution model described in the next chapter, since its diameter distribution statistics were derived by disaggregating these stand values.

CHAPTER 5 GROWTH AND YIELD MODEL DFCNIGM3 ____ DIAMETER DISTRIBUTION MODEL COMPONENTS

This chapter describes diameter distribution model components of DFCNIGM3, the acronym given for the Douglas Fir Central North Island Growth Model version 3, that is totally compatible with the stand level model presented in Chapter 4. The diameter distribution model was built by using the reverse Weibull distribution function, type III Extreme Value Distribution and an extreme percentile. The reverse Weibull b and c parameters were calculated in the usual manner; the "a" parameter was estimated through an extreme percentile on the type III extreme value distribution, whose parameters were projected for future stand conditions just as was done for the initial distribution. The extreme percentile was determined on the basis of return period. A modified height equation that reflects site variation was used to estimate height corresponding to the mid-point diameters. This chapter describes these functions in more detail.

5.1 THE WEIBULL DIAMETER DISTRIBUTION MODEL

5.1.1 The Weibull Distribution

The Weibull distribution is named after Waloddi Weibull, a Swedish physicist, who first used it to represent the relative frequency of the breaking strength of materials (Weibull, 1939). The distribution was introduced to forest diameter distribution modelling by Bailey in 1972 and further discussed by Bailey and Dell in 1973. It has become the most popular distribution function used in recent diameter distribution growth and yield modelling mainly because it has an analytic closed form c.d.f. The distribution function is given by

$$f(x)=\frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]\ldots\ldots\ldots\ldots\ldots(5.1.1.1a)$$

with cumulative distribution function.........(5.1.1.1*b*)

$$F(x)=1-\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.1.1.2)$$

Where

a = location parameter ;

b = scale parameter ;

c = shape parameter;

x = a random variable.

There are several methods of estimating parameters a, b, and c of the distribution. For example, maximum likelihood, method of moments, percentile method, estimators based on distribution of log(x) and estimators based on order statistics (Johnson and Kotz, 1970; Cohen 1965). The first three are the ones most often used in forest diameter distribution modelling (Bailey and Dell, 1973).

The maximum likelihood method is regarded as the one that gives the best estimate on minimum variance grounds, but it requires much computational effort. The method of moments is the second most efficient and the percentile third. In practice, all three methods give very similar estimates (Woollons and Whyte, pers. comm.). In this study, the method of moments was used because it is relatively efficient and easier to compute. The standard deviation (s) and the mean diameter $\bar{x}$ of a stand can be estimated by projection equations. The c parameter can then be estimated through

$$\frac{s}{\bar{x}}=\frac{\left(\Gamma(1+\frac{2}{c})-\Gamma^2(1+\frac{1}{c})\right)^{\frac{1}{2}}}{\Gamma(1+\frac{1}{c})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.1.1.3)$$

using an iterative procedure involving Γ, the gamma function. The parameter b can be calculated by solving the following:

$$\bar{x}=b^{\frac{1}{c}}\Gamma(1+\frac{1}{c}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.1.1.4)$$

In the conventional method, the "a" parameter is estimated as a function of stand attributes or set to a fixed value. In this study it was estimated from the type III extreme value distribution in combination with an extreme percentile.

5.1.2 Shortcomings of the Conventional Diameter Distribution Model Using Weibull

As mentioned above, the b and c parameters of the Weibull distribution can be estimated by the method of moments using (5.1.1.3) and (5.1.1.4). Those moments can be estimated by equations that have a reasonably good fit. For example, Matney et al., (1987) reported the R^2 for arithmetic mean diameter and quadratic mean diameter were 0.8757 and 0.9954 respectively. But the a parameter so far has been poorly estimated or determined arbitrarily. Bailey suggested that "a" be set equal to the minimum diameter of the stand (Bailey, 1972), Knoebel et al., (1986) set it equal to one half of the minimum diameter of the stand. In summary, shortcomings of the conventional Weibull diameter distribution model are:

1. the location parameter, a, has rarely been estimated satisfactorily because small trees in a stand are significantly influenced by genetic, microsite, and silvicultural factors (see Clutter and Belcher 1978);

2. some researchers set the location equal to an arbitrarily chosen value;

3. even if "a" is estimated satisfactorily by a prediction equation, it might not be close to the true minimum diameter of the stand because of the limitations placed on sampling in the stands (i.e. a limited number of observations may not encounter the minimum diameter of the stand).

These problems can be solved by using the reverse Weibull distribution and extreme value distribution. Kuru et al., (1990) used the reverse Weibull distribution and type I extreme value distribution in their study. The use of reverse Weibull distribution was adopted in this study, but it is shown that the type III extreme value distribution should be used when the initial distribution is the reverse Weibull. In addition, an extreme percentile was used in combination with the extreme value distribution to estimate the "a" parameter, which improved the goodness-of-fit for the diameter distribution model considerably.

5.2 THE REVERSE WEIBULL DIAMETER DISTRIBUTION MODEL

5.2.1 The Reverse Weibull Distribution

The Reverse Weibull distribution function is given by

$$f(x)=\exp[-\frac{a-x}{b})^{c}] \qquad -\infty \le x \le a \ldots\ldots\ldots\ldots (5.2.1.1a)$$

$$=1 \qquad x \ge a \ldots\ldots\ldots\ldots\ldots\ldots (5.2.1.1b)$$

with density

$$f(x)=\frac{c}{b}[(\frac{a-x}{b})^{c-1}]\exp[-(\frac{a-x}{b})^{c}] \ldots\ldots\ldots\ldots\ldots (5.2.1.2)$$

The distribution has the same parameter estimation equation as for the Weibull distribution (Gumbel, 1958):

$$\frac{s}{\bar{x}}=\frac{(\Gamma(1+\frac{2}{c})-\Gamma^{2}(1+\frac{1}{c}))^{\frac{1}{2}}}{\Gamma(1+\frac{1}{c})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.2.1.3)$$

$$\overline{x}=b^{\frac{1}{c}}\Gamma(1+\frac{1}{c}) \quad \text{.................................} (5.2.1.4)$$

Symbols in (5.2.1.1) to (5.2.1.4) are the same as defined previously.

The advantage of using the reverse Weibull distribution is that it enables one to use the maximum diameter as a location, which is much easier to fit.

In estimating the reverse Weibull parameters, quadratic mean diameter of the stand is obtained by solving

$$d_g=200\sqrt{\frac{G}{\pi N}} \quad \text{.................................} (5.2.1.5)$$

and arithmetic mean can be calculated through

$$s^2=\frac{\sum(d_g^2)-(\sum d)^2/N}{N} \quad \text{..........................} (5.2.1.6)$$

Rearranging (5.2.1.6) we have

$$s^2N=\frac{N^2\sum(d_g^2)}{N^2}-\frac{N(\sum d)^2}{N^2}$$

which is equivalent to

$$s^2=\frac{\sum(d_g^2)}{N}-(\frac{\sum d}{N})^2$$

Obviously

$$\frac{\sum(d_g^2)}{N}=d_g^2, \qquad (\frac{\sum d}{N})^2=d^2$$

Thus we arrive at

$$d^2 = d_g^2 - S^2 \qquad (5.2.1.7)$$

The reverse Weibull b and c parameters can then be solved through (5.2.1.3) and (5.2.1.4) by substituting the standard deviation and mean diameter for s and $\bar{x}$. The a parameter is not directly set equal to the projected maximum diameter; rather, it is estimated through type III Extreme Value Distribution in combination with an extreme percentile. Estimating parameter a in this way ensures not only that the location is or close to the maximum diameter but also this maximum diameter will not be exceeded at a specified age. The purpose of fitting the maximum diameter projection equation below is to demonstrate how much easier it is to fit when one works with maximum diameter than the minimum.

5.2.2 Maximum Diameter Projection Equation

Diameters at breast height of all trees in a PSP plot are usually measured. The maximum diameter of a plot observed each time was extracted and the Hossfeld equation (Woollons, Whyte and Liu Xu 1990) was fitted to those observations.

$$d_{max2} = \frac{1}{\frac{1}{d_{max1}}\left(\frac{T_1}{T_2}\right)^{\gamma} + \alpha\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma}\right) + \beta X\left(1-\left(\frac{T_1}{T_2}\right)^{\gamma}\right)} \qquad (5.2.2.1)$$

Table (5.2.2.1) shows the estimated statistics of this equation for (a) diseased, (b) healthy plots respectively. Group (c) is the same equation fitted to the minimum diameter of diseased plots.

Table (5.2.2.1) Estimated statistics of maximum diameter Projection equation

set	Estimates and STD error of			n
	α	β	γ	(M. Res)
a	1.325263557	0.008122609	0.002267063	1662
	0.019122020	0.000171839	0.000216616	0.006106
b	1.452939884	0.008481616	0.001832766	116
	0.151967126	0.001181885	0.000701823	0.001204
c	2.339830340	0.019528869	0.010413139	1662
	0.163045145	0.001353985	0.003814423	0.276305

For each set in the table, the first row corresponding to α, β and γ estimates while the second row is the corresponding standard error of the estimated parameters. The last column refers to the sample size and mean sums of squares of residuals respectively. When the same equation is fitted to the minimum diameter, the standard error of the three estimates increased by 7, 6.8, 16.6 times and the mean residual sums of squares increased by 44.3 times in relation to that from the maximum diameter. Fig. (5.2.2.1) to (5.2.2.4) are graphs of residuals and residual bar charts for the maximum diameter equation for the diseased and undiseased stands respectively. Residuals and bar charts for the minimum diameters are given in Fig (5.2.2.5) and (5.2.2.6). As can be seen, for the same data set and same equation, maximum diameter produced a much better result.

RESIDUAL PLOT OF MAXIMUM DIAMETER EQUATION

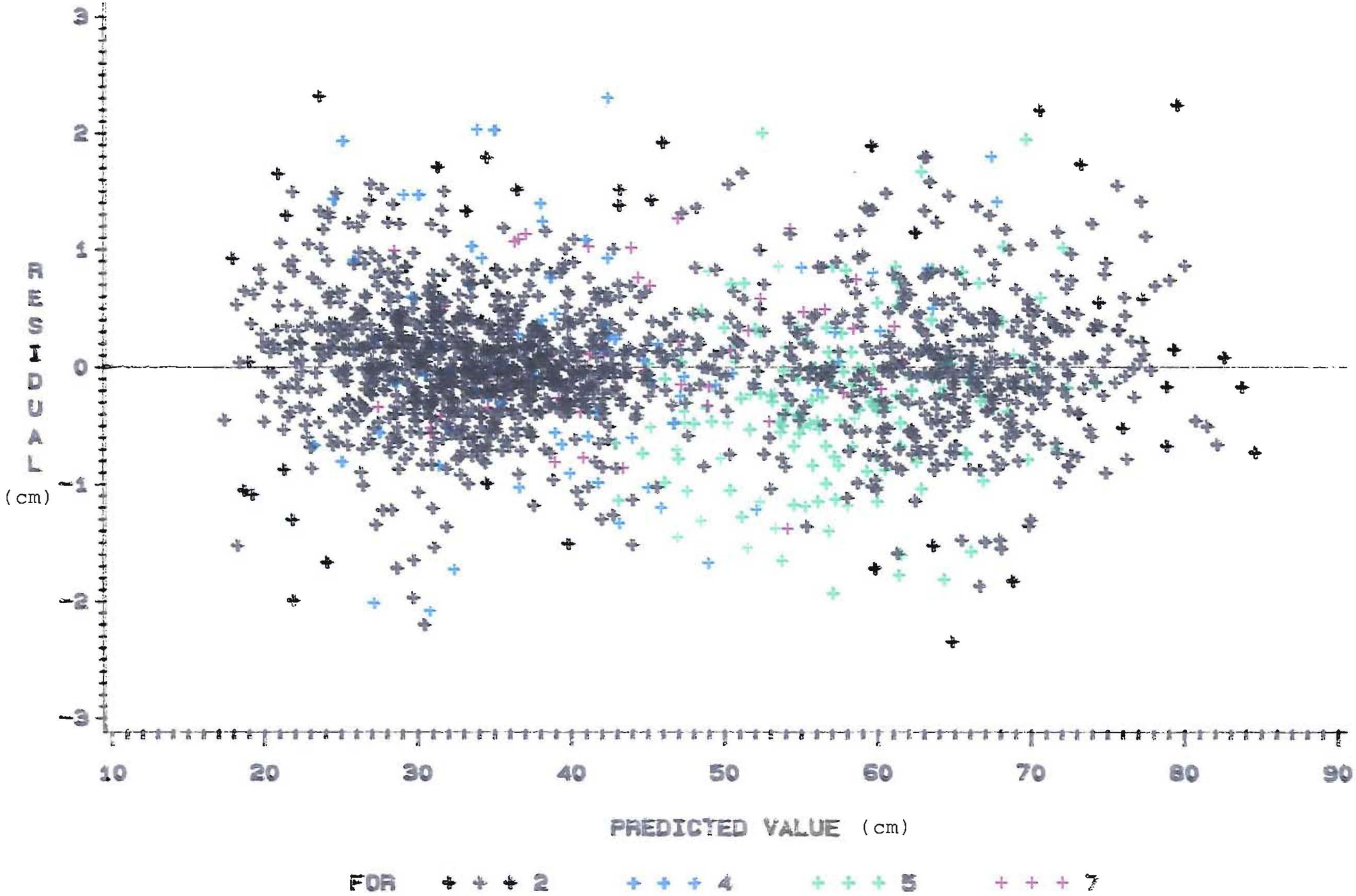


FIG (5.2.2.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF MAXIMUM DIAMETER EQUATION

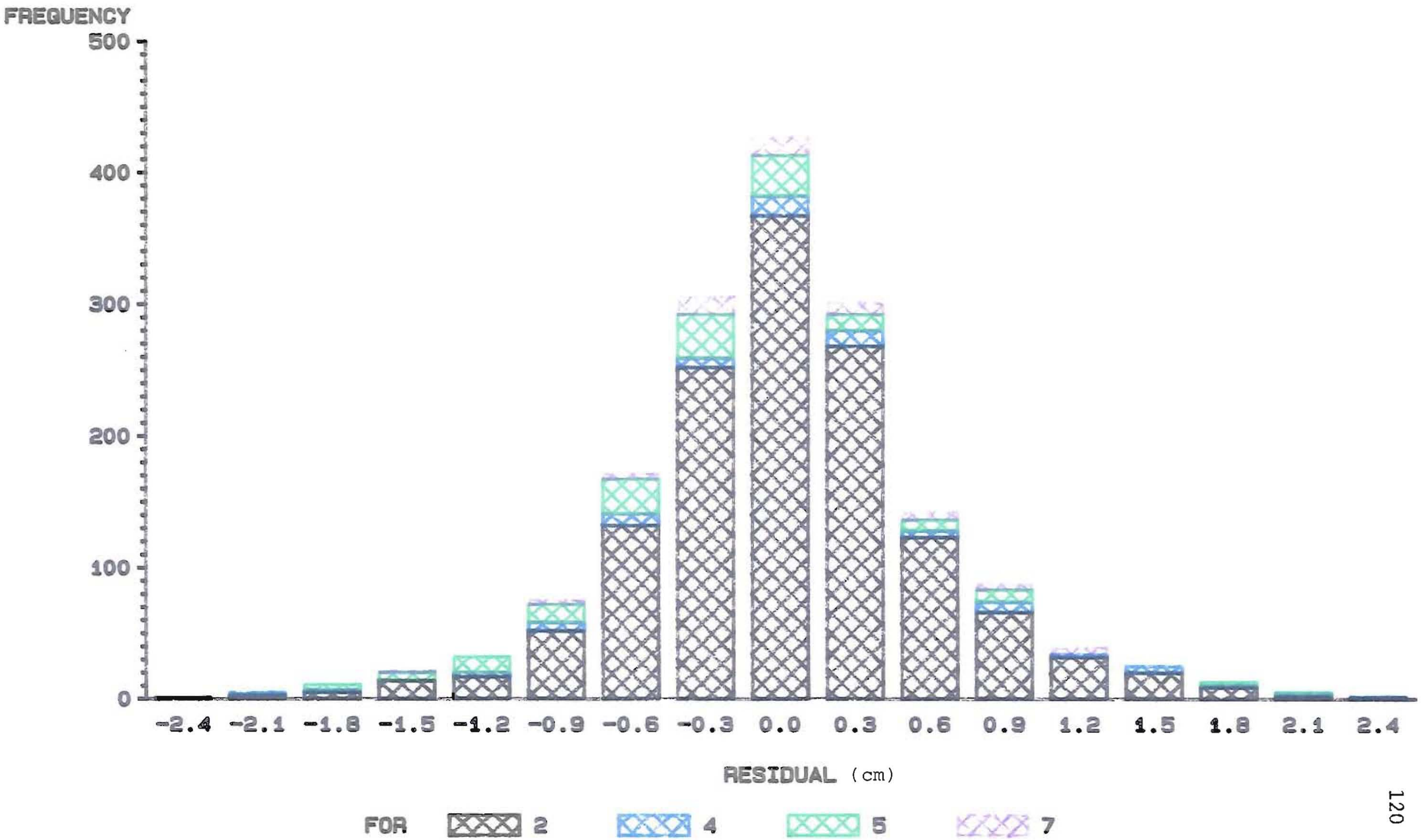


FIG (5.2.2.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF MAXIMUM DIAMETER EQUATION

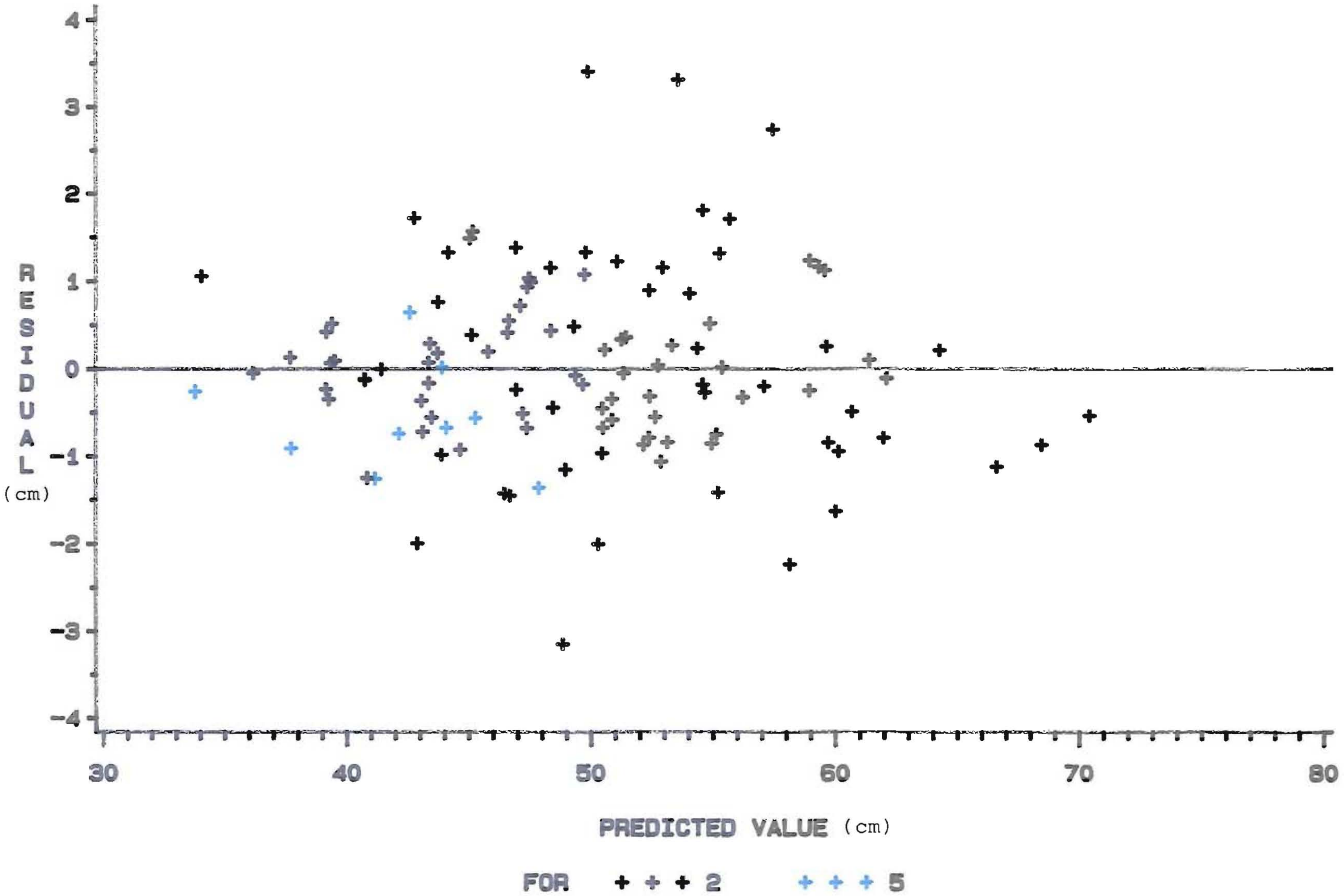


FIG (5.2.2.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF MAXIMUM DIAMETER EQUATION

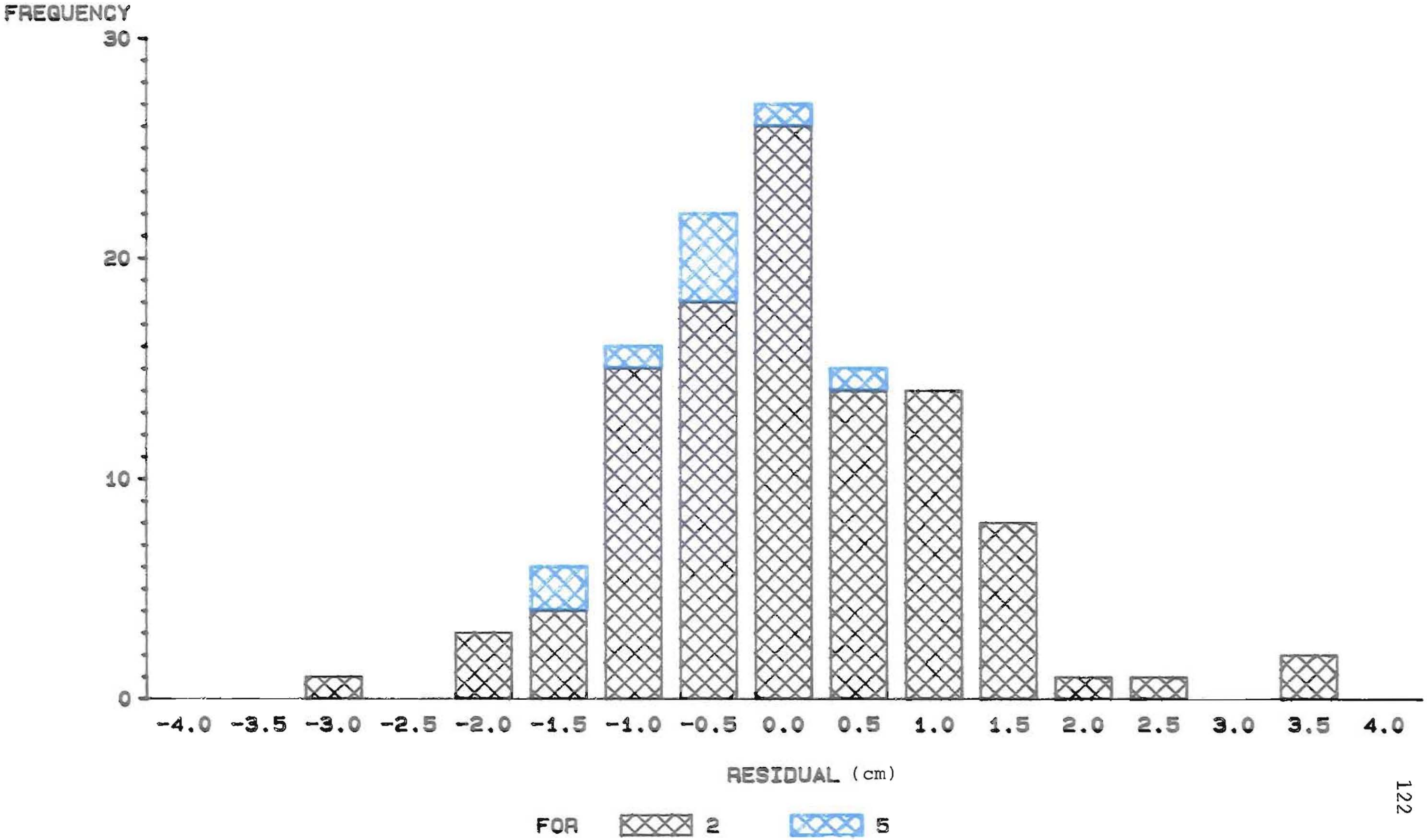


FIG (5.2.2.4): KAINGAROA UNDISEASED STANDS

RESIDUAL PLOT OF MINIMUM DIAMETER EQUATION

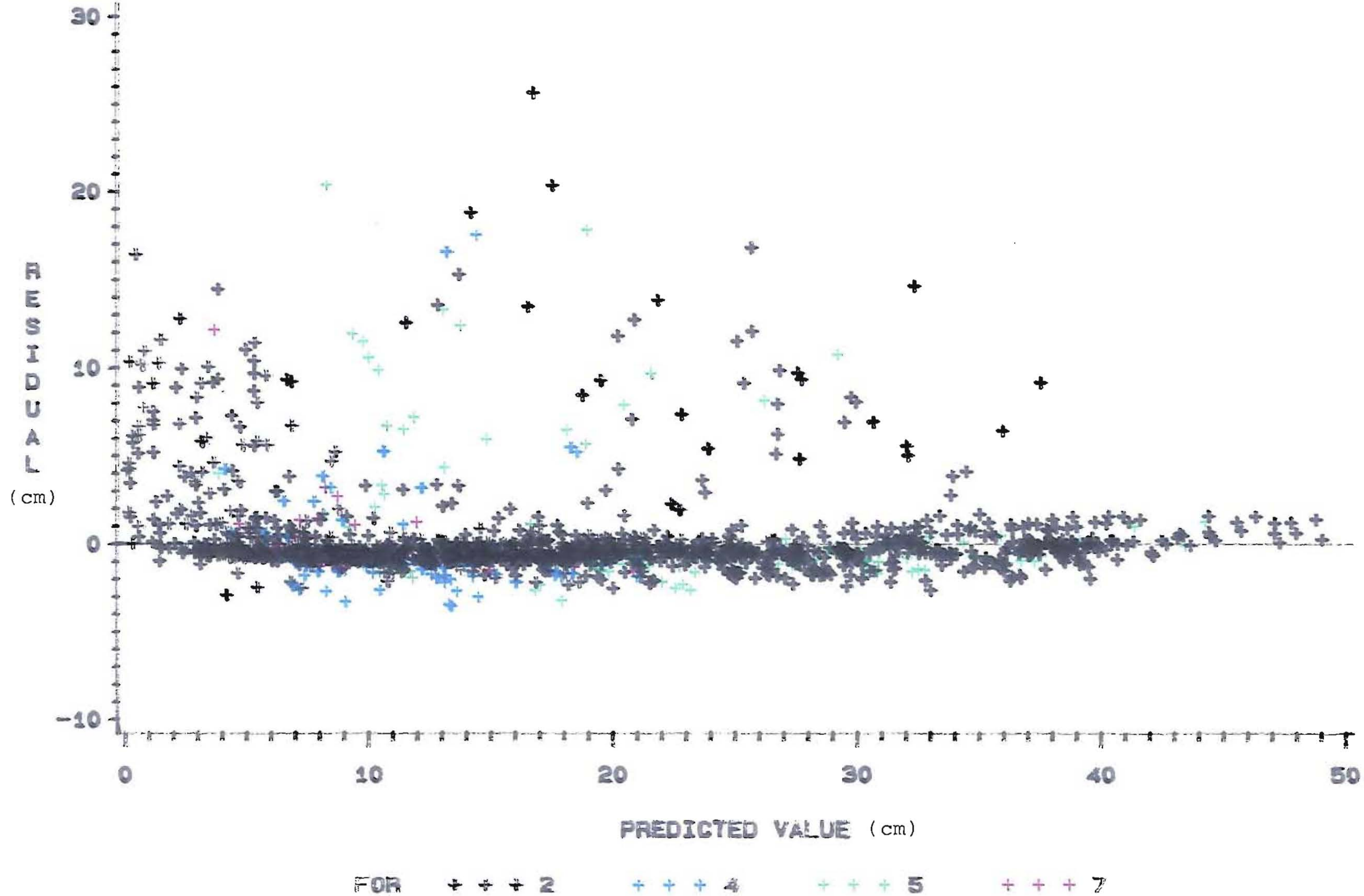


FIG (5.2.2.5): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF MINIMUM DIAMETER EQUATION

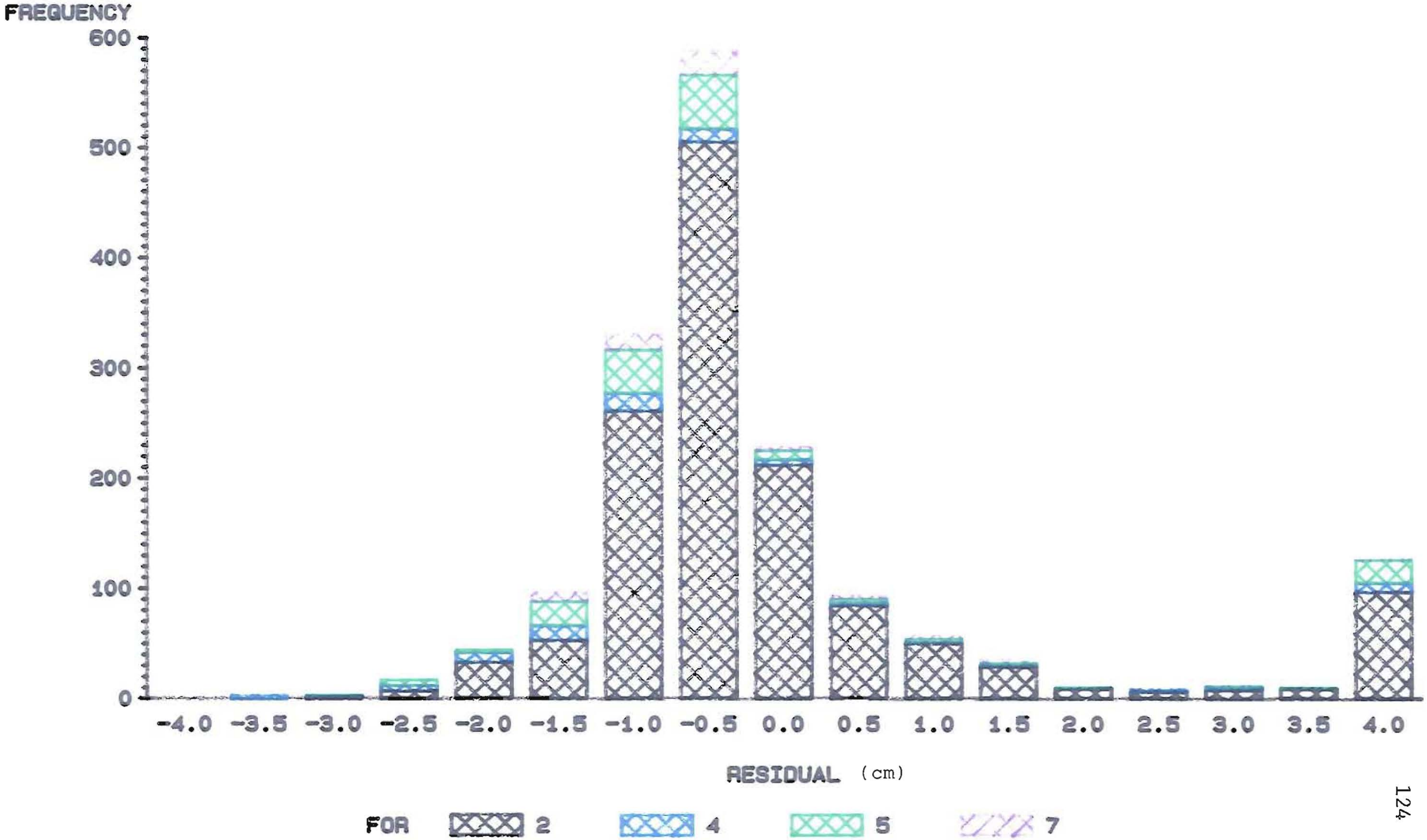


FIG (5.2.2.6): KAINGAROA DISEASED STANDS

5.2.3 Standard Deviation Projection Equation

Standard deviations were calculated for tree diameters in a plot observed each time and projection equations were fitted to the calculated standard deviations using the Hossfeld equation modified into

$$d_{s2}=\frac{1}{\frac{1}{d_{s1}}\left(\frac{T_1}{T_2}\right)^{\delta}+\alpha d_{max1}\left(1-\left(\frac{T_1}{T_2}\right)^{\delta}\right)+\beta d_{min1}\left(1-\left(\frac{T_1}{T_2}\right)^{\delta}\right)+\gamma k\left(1-\left(\frac{T_1}{T_2}\right)^{\delta}\right)}$$

..........................(5.2.3.1)

Where $k = \log((G_2-G_1)/(T_2-T_1))$, d_{s1} and d_{s2} are standard deviations of the diameter. Other variables in the equation have been previously defined. Table (5.2.3.1) shows the estimated statistics for the standard deviation projection equation for set a (diseased plots) and b (healthy plots).

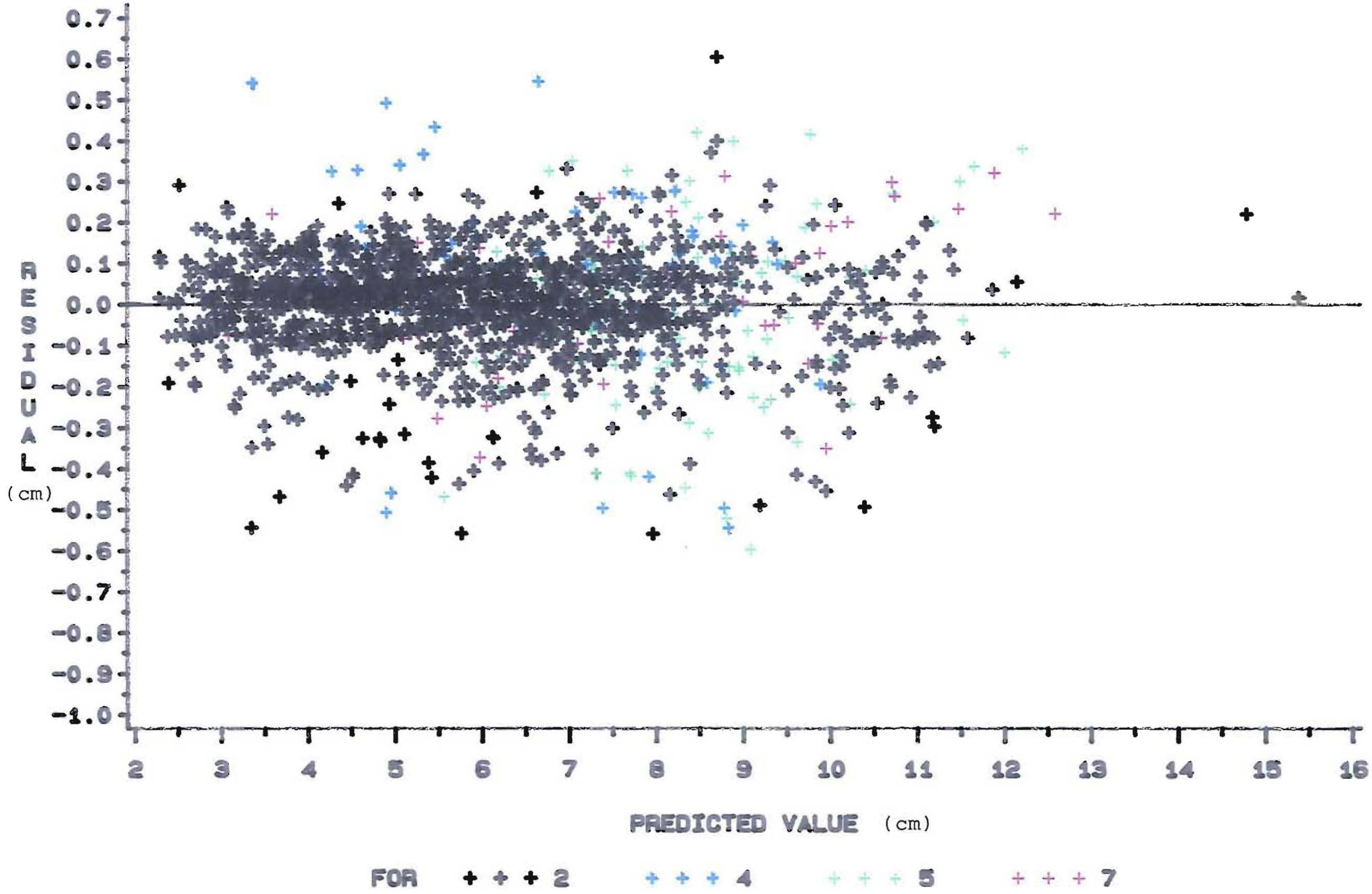
Table (5.2.3.1) Estimated statistics of the standard deviation projection equation

set	Estimates and STD error of				n
	α	β	γ	δ	
a	0.9044947 0.0274059	0.0019577 0.0009955	-0.00364 0.000241	-0.06247 0.007442	1451
b	1.2794697 0.0952801	0.0011342 0.0002066	-0.00072 0.000668		114

Fig. (5.2.3.1) to (5.2.3.4) are the graphs of residual and residual bar chart for this equation for the diseased and healthy plots respectively.

At this point, the Weibull parameters b and c are ready to be solved: d_g, d and s can be solved through equation (5.2.1.5), (5.2.1.7) and (5.2.3.1) respectively. Then the parameter c can be solved through equation (5.2.1.3) with d in place of $\bar{x}$ and parameter b can be solved through (5.2.1.4). The Weibull a parameter is to be solved through using the type III extreme value distribution and an extreme percentile, which will be described in sections 5.3 to 5.5.

RESIDUAL PLOT OF STANDARD DEVIATION EQUATION



RESIDUAL CHART OF STANDARD DEVIATION EQUATION

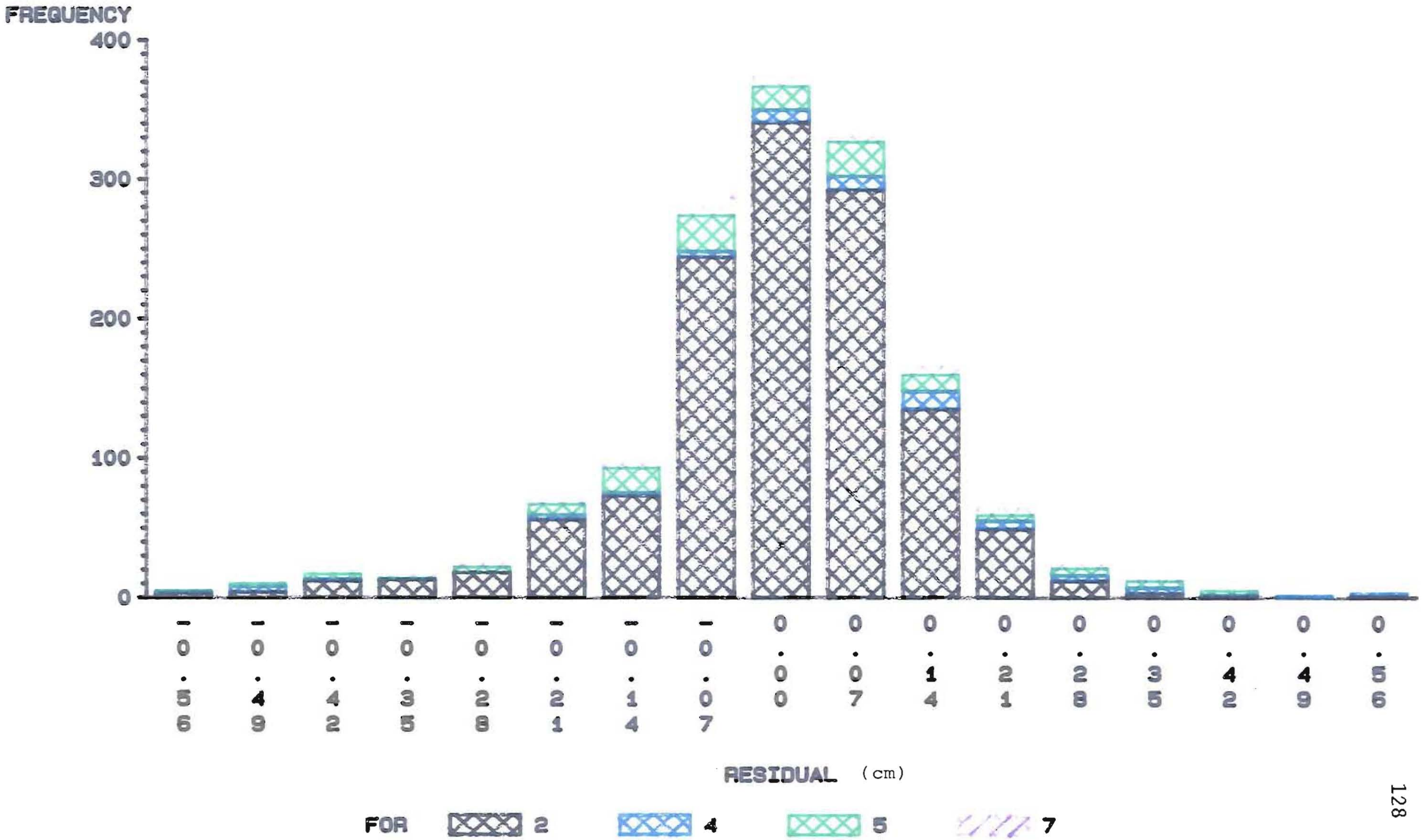


FIG (5.2.3.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF STANDARD DEVIATION EQUATION

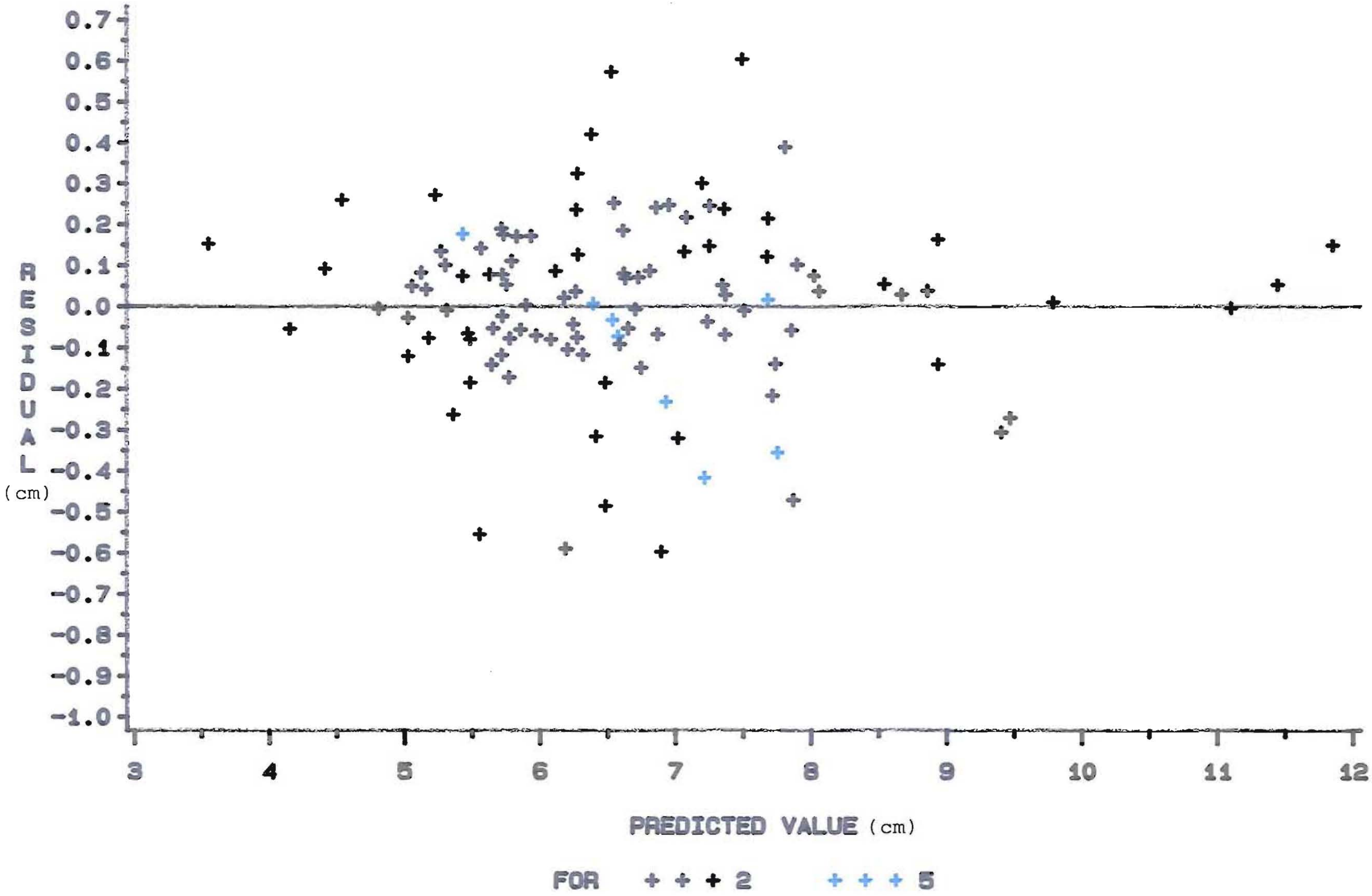


FIG (5.2.3.3): KAINGAROA UNDISEASED STANDS

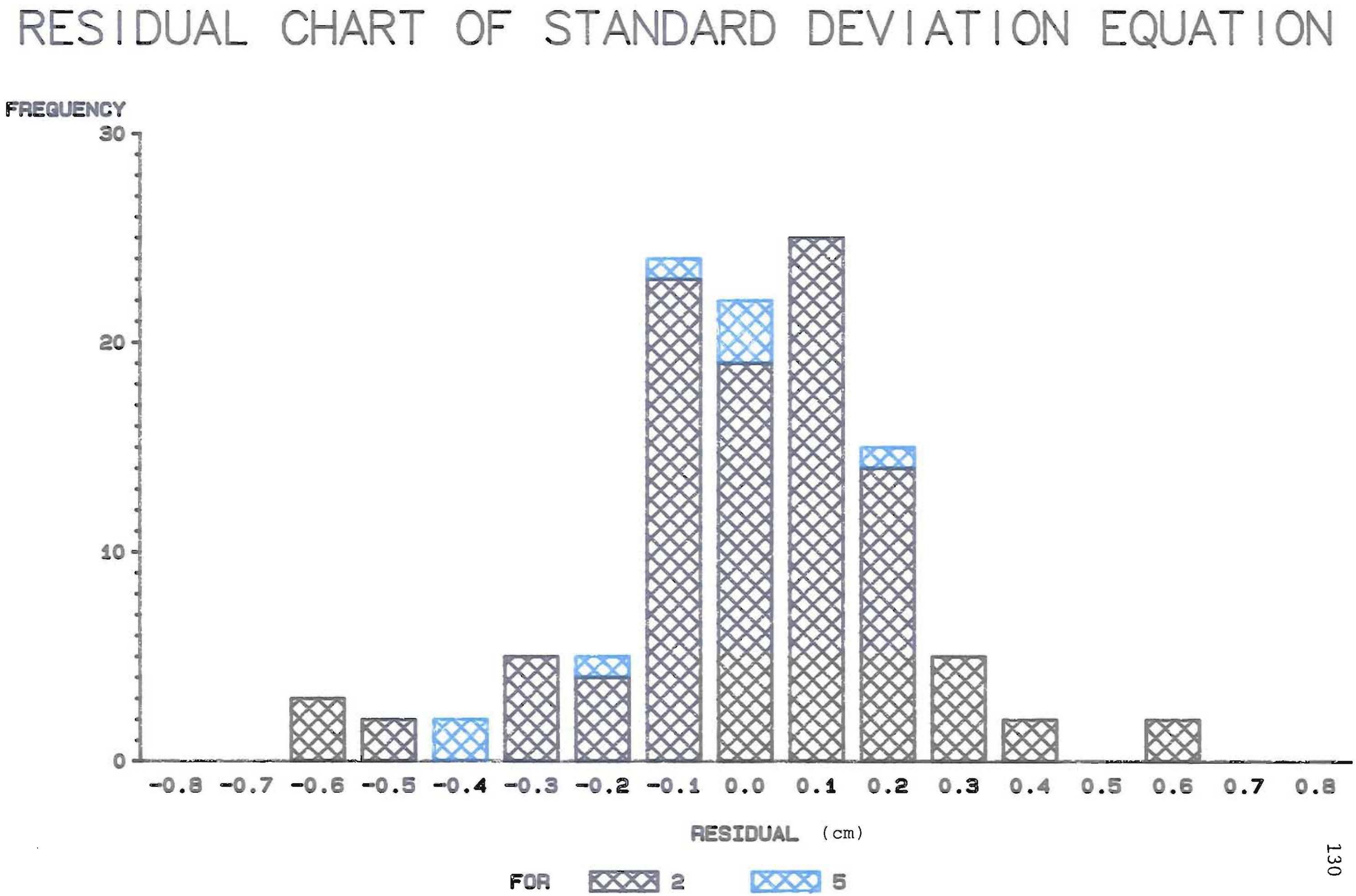


FIG (5.2.3.4): KAINGAROA UNDISEASED STANDS

5.3 THE USE OF TYPE III EXTREME VALUE DISTRIBUTION AND THE EXTREME PERCENTILE

In this section, a procedure based on the type III extreme value distribution and an extreme percentile for estimating the Reverse Weibull a parameter was proposed. It possesses the following new features:

(1) **Use of type III extreme value distribution.** When the Reverse Weibull distribution is used as the initial distribution the extreme value distribution should be the type III rather than type I. This is proved mathematically on the basis of Stability Postulate (Fréchet, 1927; Fisher and Tippett, 1928) and Gnedenko conditions (Gnedenko, 1943).

(2) **Use of am extreme percentile.** An extreme percentile was chosen from the extreme value distribution to eliminate biases that can occur when the percentile is chosen arbitrarily (Gumbel, 1958);

(3) **Parameter projection of the type III extreme value distribution.** The parameters of the extreme value distribution are projected so that any forecast made is derived from a reverse Weibull distribution and parameter a of this distribution is estimated from the type III Extreme Value Distribution. The choice of the percentile on the extreme value distribution for calculating the "a" parameter was made on the basis of an extreme percentile calculated from the return period (Gumbel, 1958). This procedure for calculating the reverse Weibull "a" ensures

that the "a" parameter chosen is or is close to the true maximum diameter of the stand, not to be exceeded for a specified stand age.

The procedure is described in more detail in the following subsections.

5.3.1 A Brief Review of the Extreme Value Theory

As early as 1709 Nicolaus Bernoulli considered this problem: n men of equal age die within t years. What is the mean duration of life of the last survivor? This is in fact a problem of the largest value (Gumbel, 1958). In 1852, Peirce reported criteria for the acceptance or rejection of an outlying observation, which is also a problem of the largest values.

Systematic development of the theory was started in 1922 when the "distribution of largest value" was clearly introduced by Bortkiewicz (1922). In the following year, Mises introduced the fundamental notion of the characteristic largest value and evaluated the expected value of this distribution. Dodd (1923) calculated its median, and also considered some non-normal parent distribution. Tippett (1925) calculated the probabilities of the largest normal values for different sample sizes up to 1000 and the mean normal range for samples from 2 to 1000.

Most of above mentioned studies were concerned with the normal distribution. Fréchet (1927) first studied an initial distribution that is different from the normal distribution and obtained the second asymptotic distribution of the largest value. He introduced the stability postulate according to which the distribution of the largest value should be equal to the initial one, except for a linear transformation. In 1928, Fisher and Tippett made an independent inquiry into the same problem and published the paper in which they used the same stability postulate, and found in addition to Fréchet's asymptotic distribution, the first and third asymptotic distribution. Mises (1936) classified initial distributions possessing asymptotes for the largest value, and gave sufficient conditions under which the three asymptotic distributions are valid. Gumbel (1937) studied radioactive emissions using the extreme value distribution. In 1939, Weibull effectively advocated the use of reverse type III extreme value distribution which have now become well-known as the Weibull distributions (Johnson and Kotz 1970). In 1954, Leme gave a systematic exposition of the asymptotic distribution of extreme values and their application to some engineering problems.

From the late 1930's onwards, the extreme value distribution has been applied in many different fields. e.g. rainfall (Potter, 1949), flood studies (Gumbel, 1941, 1944, 1945, 1949b, Rantz and Riggs, 1949), earthquakes (Nordquist, 1945) general meteorological data (Jenkinson, 1955; Thom, 1954), aircraft load (Press, 1949), corrosion

(Aziz, 1955; Eldredge, 1957) and microorganism survival times (Velz, 1947).

5.3.2 The Stability Postulate and the Extreme Value Distribution

The stability postulate was first introduced by Fréchet in 1927. In the following year Fisher and Tippett used it in an independent study of the extreme value distributions.

The stability postulate says that the distribution of the largest value should be equal to the initial one except for a linear transformation; In this case the initial distribution is said to be stable with respect to its extreme (Gumbel, 1958). An extract of Fisher and Tippett's derivation from Gumbel (1958) is given below to explain the concept.

Consider N samples, each of size n, taken from the same population. In each sample there is a largest value and the largest value in the Nn observations is the largest of the N largest values taken from samples of size n. The distribution of the largest value in Nn observations will tend to be the same asymptotic expression as the distribution of the largest value in sample of size n, provided that such an asymptote exists. Consequently, the asymptote must be such that the largest value of a sample of size n taken from it must have the same asymptotic distribution.

Since a linear transformation does not change the form of the distribution, the probability that the largest value is below x should be equal to the probability of a linear function of x,

$$F^n(x) = F(a_n x + b_n) \quad \text{..........................(5.3.2.1)}$$

the two parameters a_n and b_n being functions of n. Equation (5.3.2.1) is called the Stability Postulate.

If a_n in (5.3.2.1) differs from unity, the two curves $F^n(x)$ and $F(x)$ are not parallel. There is a value x' where the two probabilities are equal. This value is

$$x_n' = a_n x' + b_n; \qquad x' = b_n / (1-a_n)$$

The equation $F^n(x') = F(x')$ can be satisfied if and only if

$$F(x') = 0 \text{ or } F(x') = 1 \quad \text{.....................(5.3.2.2)}$$

If we shift the origin of the variate, we may put $x' = 0$, hence $b_n = 0$. The third asymptotic distribution of the largest values is derived under the assumptions of $b_n = 0$ in (5.3.2.1) and $F(x') = 1$ in (5.3.2.2):

$$F^n(x) = F(a_n x) \quad \text{............................(5.3.2.3)}$$

Two functions need to be determined: a_n as a function of n and the initial probability $F(x)$. Raising F to the mth power is the same as multiplying x by a_m. Therefore:

$$[F^n(x)]^m = F^{nm}(x) = F(a_n a_m x)$$

On the other hand and from (5.3.2.3)

$$F^{nm}(x) = F(a_{nm}x) \quad \text{..............................(5.3.2.4)}$$

by combining the preceding two equations, we have

$$a_{nm} = a_n . a_m \quad \text{..................................(5.3.2.5)}$$

Its solution is

$$a_n = n^L \quad \text{.......................................(5.3.2.6)}$$

To obtain the probability function F(x), we take logarithms of (5.3.2.3) and introducing (5.3.2.6), which leads to

$$\log(n) + \log\{-\log[F(x)]\} = \log\{-\log[F(n^L x)]\}$$

which shows that if log(x) is increased by $\log(a_n)$, then log{-log[F(x)]} is increased by log(n) so that

$$\log\{-\log[F(x)]\}-\log(x)/L = \text{constant} \quad \text{...........(5.3.2.7)}$$

This expression can be changed into

$$\log\{-\log[F(x)]\} = [\log(x)-\log(v)]/L$$

where v is transformation of the constant in (5.3.2.7). Consequently,

$$-\log[F(x)] = (x/v)^{1/L};\ F(x) = \exp[-x/v)^{1/L} \quad \text{.....(5.3.2.8)}$$

If $F(x') = 1$, i.e., if the variate is non-positive, the factor in (5.3.2.8) is positive, and $1/L = k > 0$, the third initial probability function which is stable with respect to the largest value is obtained:

$$F^{n}(x) = \exp[-(xn^{1/k}/c)^{k}] \quad \text{......................(5.3.2.9)}$$

This function is the reverse Weibull function (Fréchet, 1927; Fisher and Tippet, 1928; Gumbel, 1958; Johnson and Kotz, 1970).

As shown by the Stability Postulate given above, this distribution is stable with respect to its largest value. In other words, the largest value distribution of the reverse Weibull distribution should be the reverse Weibull distribution itself.

It is possible for the same initial distribution to have different extreme value distribution (Gumbel, 1958). Gnedenko (1943) gave the necessary conditions for the existence of the first, second and third asymptotic distributions of largest values. Those conditions are:

(1) *for* *I*: $\lim_{n=\infty} n[1-F(u_n+y/\alpha_n)]=e^{-y}$.......(5.3.2.10)

(2) *for* *II*: $\lim_{x=0}\frac{1-F(x)}{1-F(cx)}=c^k$.............(5.3.2.11)

(3) *for* *III*: $\lim_{x=-0}\frac{1-F(cx+\omega)}{1-F(x+\omega)}=c^k$.........(5.3.2.12)

When the initial distribution used is any one of the three types of extreme value distributions, the extreme value distribution is the same as the initial distribution and the correctness of such a usage need not to be tested according to the Stability Postulate. When the initial distribution used is not any one of the three types of asymptotic distributions, then the existence of such an extreme value distribution should be tested by the Gnedenko conditions given above.

In this study, the initial distribution used is the reverse Weibull distribution, which belongs to type III asymptote; thus the extreme value distribution is the Reverse Weibull itself and need not be tested. However, The test is carried out below to demonstrate the procedure. In the standard form with a = 0, F(x) = exp[-(-x/b)^c]:

$$\lim_{x\to-0}\frac{1-F(\alpha x)}{1-F(x)}=\lim_{x\to-0}\frac{1-\exp[-(\frac{-\alpha x}{b})^c]}{1-\exp[-(\frac{-x}{b})^c]} \quad \textit{let } z = x/b$$

$$=\lim_{Z\to-0}\frac{1-\exp[-(-\alpha Z)^c]}{1-\exp[-(-Z)^c]}$$

$$=\lim_{Z\to -0}\frac{-e^{-(-\alpha Z)^{c}}.-c(-\alpha Z)^{c-1}.-\alpha}{-e^{-(-Z)^{c}}.-c(-Z)^{c-1}.-1}$$

$$=\lim_{Z\to -0}\frac{e^{-(-\alpha Z)^{c}}.\alpha^{c-1}.\alpha}{e^{-(-Z)^{c}}}$$

$$=\alpha^{c}$$

It confirms the Gnedenko conditions and thus the Reverse Weibull should be used as the extreme value distribution when Reverse Weibull is used as the initial distribution.

There is also a symmetry principle, which says that for a given distribution of largest values, valid for variate x, we may obtain a distribution of a smallest value by changing the sign of x. In two mutually symmetrical distributions, the distribution of the largest value of the one is the distribution of the smallest value of the other, and vice-versa.

All three asymptotic distribution of extreme values are linked by the symmetry principle to asymptotic distribution of smallest values (Gumbel, 1958). So, according to the symmetry principle, the Weibull distribution is the smallest extreme value distribution when one works with the smallest diameters. In other words, the procedure proposed here can be used in the conventional Weibull distribution approach. In terms of the three parameter Reverse Weibull distribution [EQ 5.2.1.1)], an expression of

$$x = a_x - b_x[-in(p))]^{1/cx} \qquad (5.3.2.13)$$

is ready obtainable. Where a_x, b_x and c_x are location, scale and shape parameters of the type III Extreme Value Distribution. This is the equation to calculate the maximum diameter of the stand based on the extreme percentile p_x on the type III Extreme Value Distribution.

5.4 PARAMETER PROJECTION OF THE EXTREME VALUE DISTRIBUTION

In this study, moments of the type III extreme value distribution were projected for future stand conditions so that the location of the reverse Weibull diameter distribution of any future stand condition is derived from the extreme value distribution. Those projection equation are described below.

5.4.1 Projection of the Mean of the Extreme

Before fitting the equation, a maximum of diameters in a plot measured at a given time was extracted. All those values extracted form a sample of extreme value, which contained in the file DMAX.SAS (Appendix 7). Then, those extreme values were sorted by age and mean value for each age was calculated. If there is only one observation for a particular age, this observation will be removed from consideration. After this organisation, the observations available for diseased and undiseased stands were 170 and 30 respectively. The Hossfeld equation was then fitted to those mean values.

$$d_{xmean2}=\frac{1}{\frac{1}{d_{xmean1}}\left(\frac{T_1}{T_2}\right)^{\beta}+\alpha\left(1-\left(\frac{T_1}{T_2}\right)^{\beta}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5.4.1.1)$$

Table (5.4.1.1) shows the estimated value of the parameters of this equation.

Table (5.4.1.1) Estimated statistics of the mean of extreme

set	Estimates of		STD error of		n
	α	β	α	β	
post -63	1.1695887	0.007650	0.045364	0.000466	170
pre-63	1.4375777	0.009196	0.139490	0.001254	30

Fig (5.4.1.1) to (5.4.1.4) are the graphs of residuals and residual bar charts for the mean of extremes for the diseased and undiseased stands.

RESIDUAL PLOT OF MEAN EXTREME EQUATION

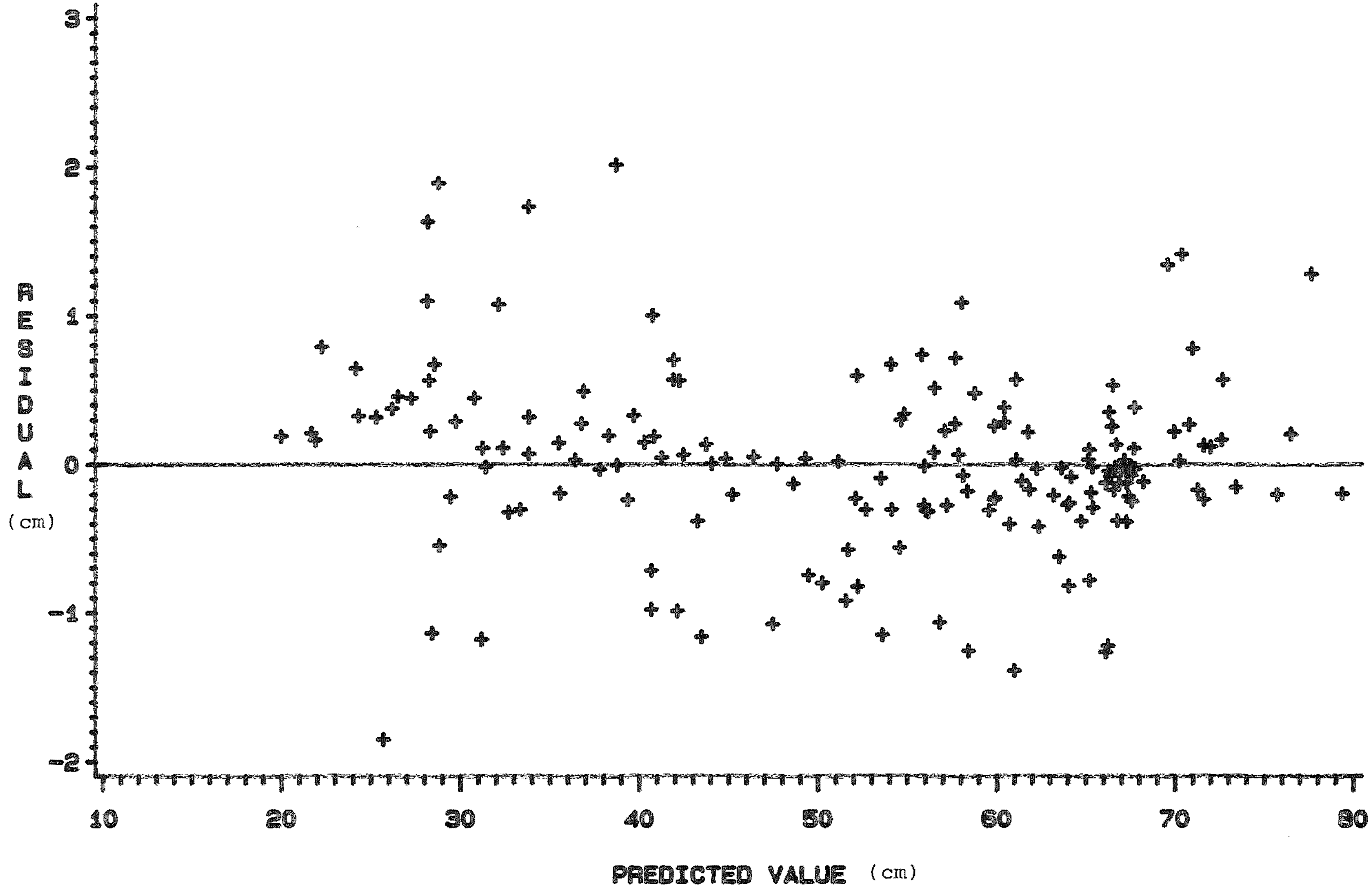


FIG (5.4.1.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF MEAN EXTREME EQUATION

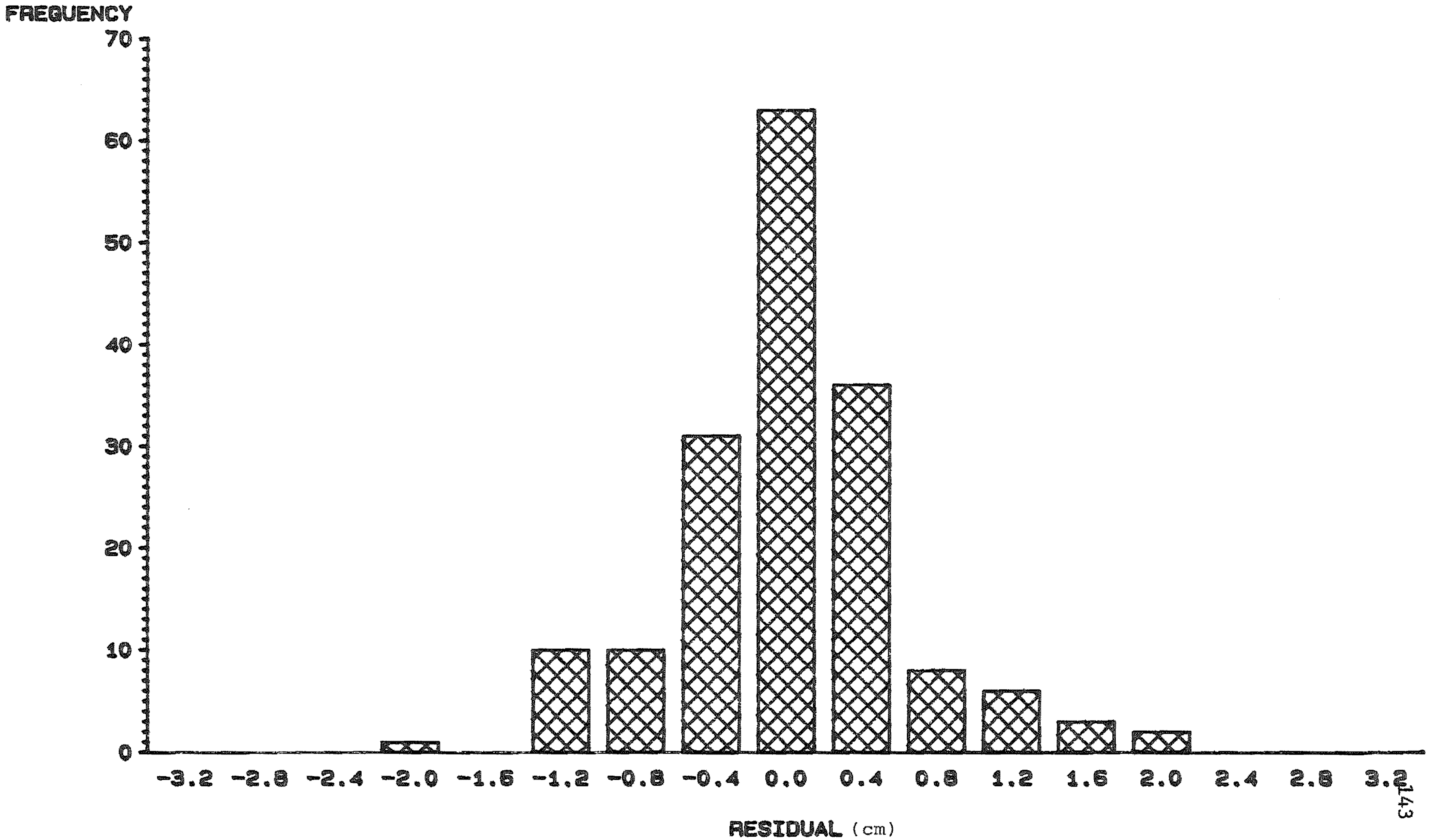


FIG (5.4.1.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF MEAN EXTREME EQUATION

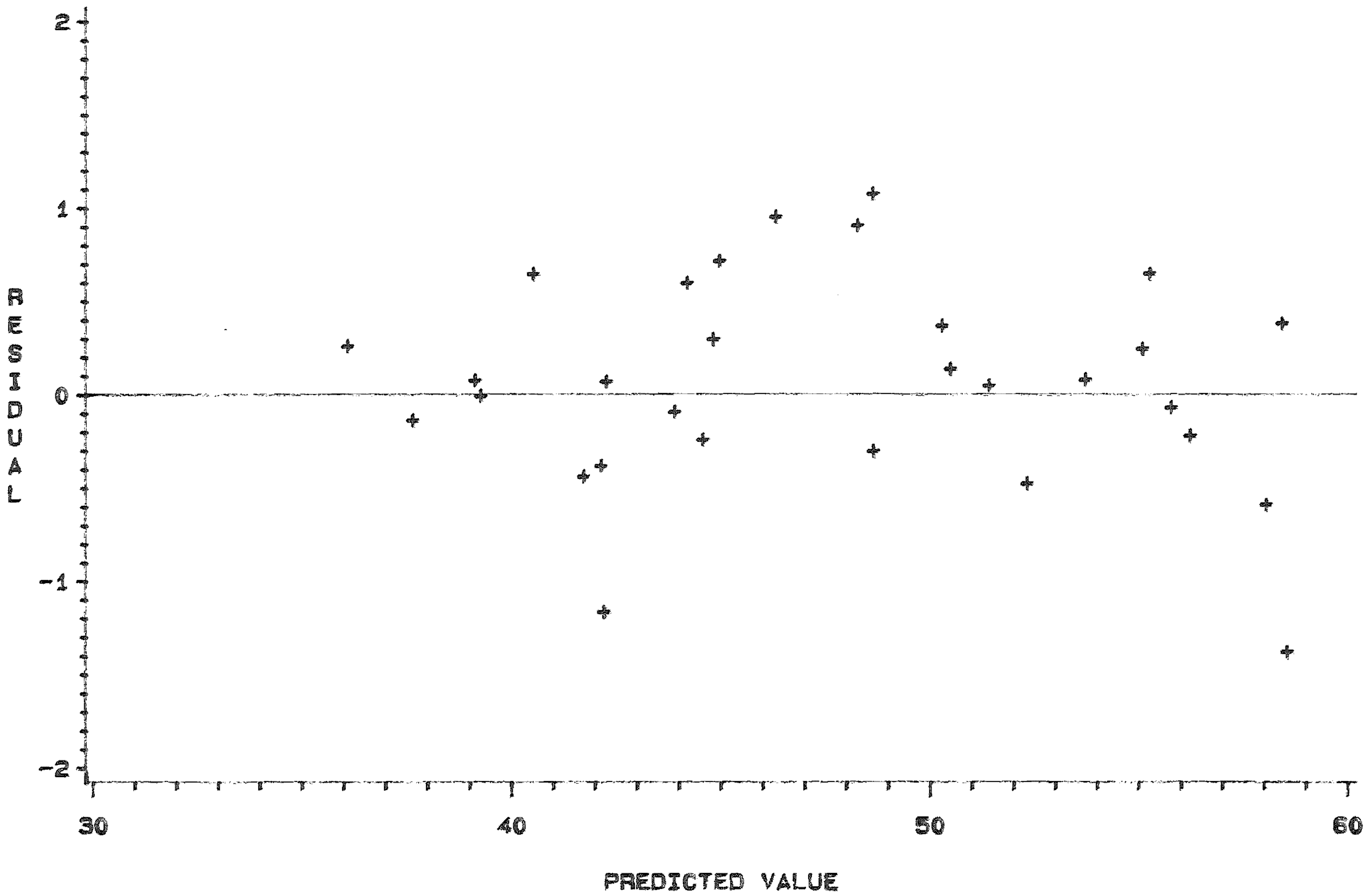


FIG (5.4.1.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF MEAN EXTREME EQUATION

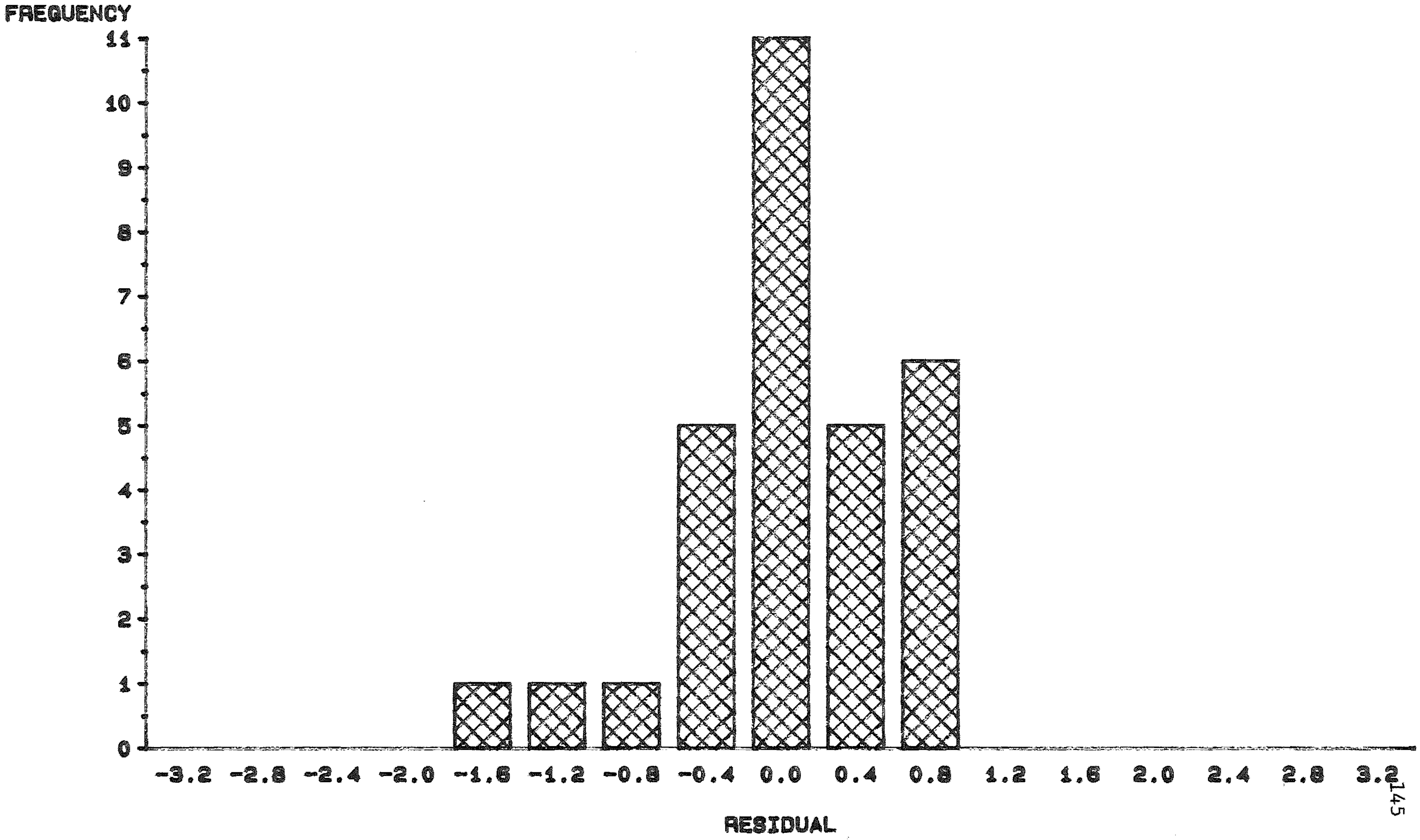


FIG (5.4.1.4): KAINGAROA UNDISEASED STANDS

5.4.2 Projection of the Standard Deviation of the Extreme

Data for fitting this standard deviation equation were formed in a similar way to that for the mean of the extreme. Values of the maximum diameters were sorted by age and a standard deviation calculated for each age. A transition equation was fitted to those values.

The equation used is the same as equation (5.4.1.1) except d_{xmean2} was replaced by d_{xstd2} and d_{xmean1} was replaced by d_{xstd1}. Table (5.4.2.1) gives the estimated statistics of this equation.

Table (5.4.2.1) Estimated statistics of the standard deviation of the extremes

set	Estimates of		STD error of		n
	α	β	α	β	
post 63	0.9982365	0.0133165	0.264261	0.033080	170
pre-63	2.2817299	0.1485426	0.957529	0.063614	30

Fig (5.4.2.1) to (5.4.2.4) are the graphs of residuals and residual charts for the standard deviation of the extreme largest diameters for diseased and undiseased stands.

RESIDUAL PLOT OF EXTREME STD EQUATION

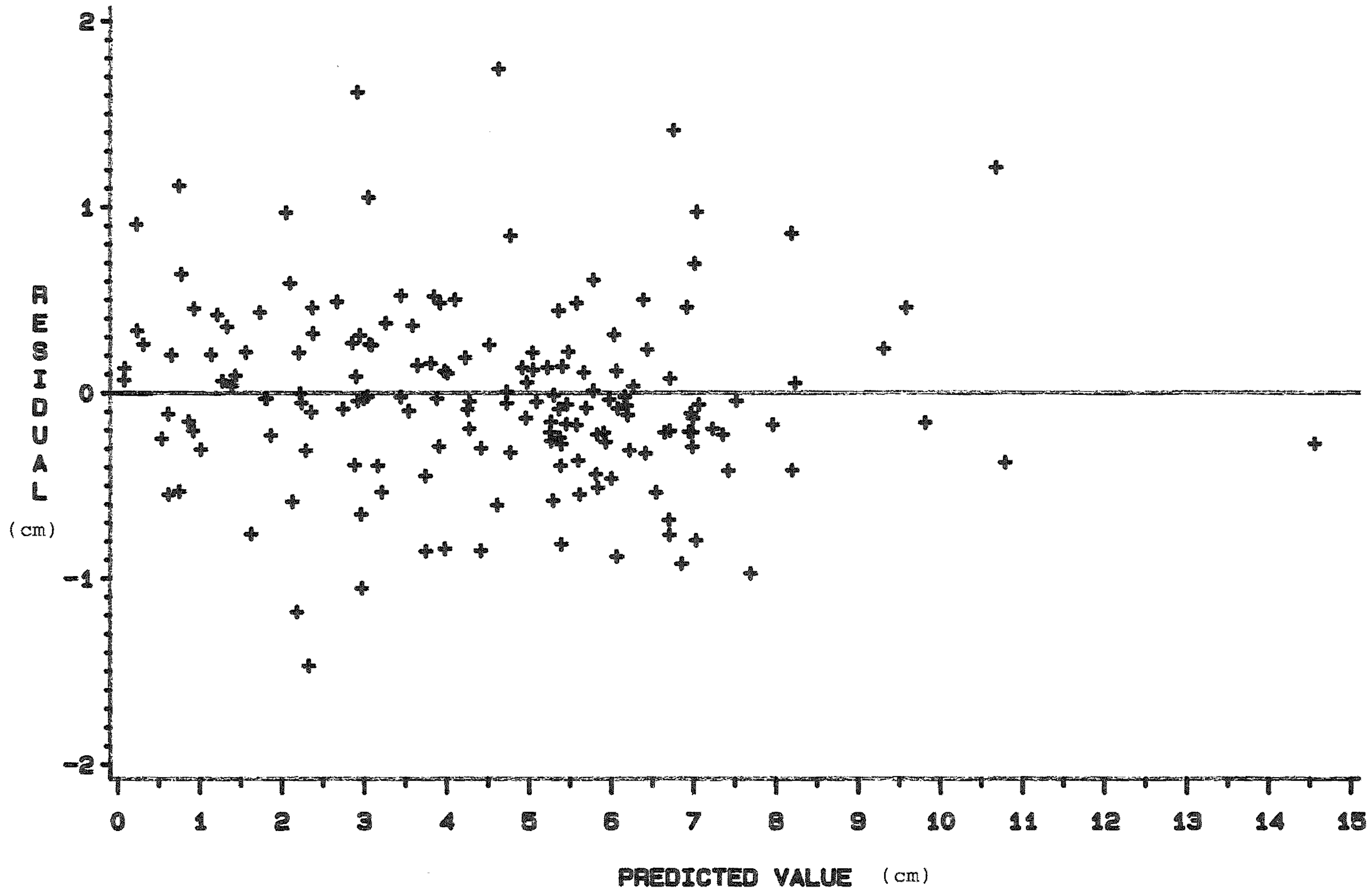


FIG (5.4.2.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF EXTREME STD EQUATION

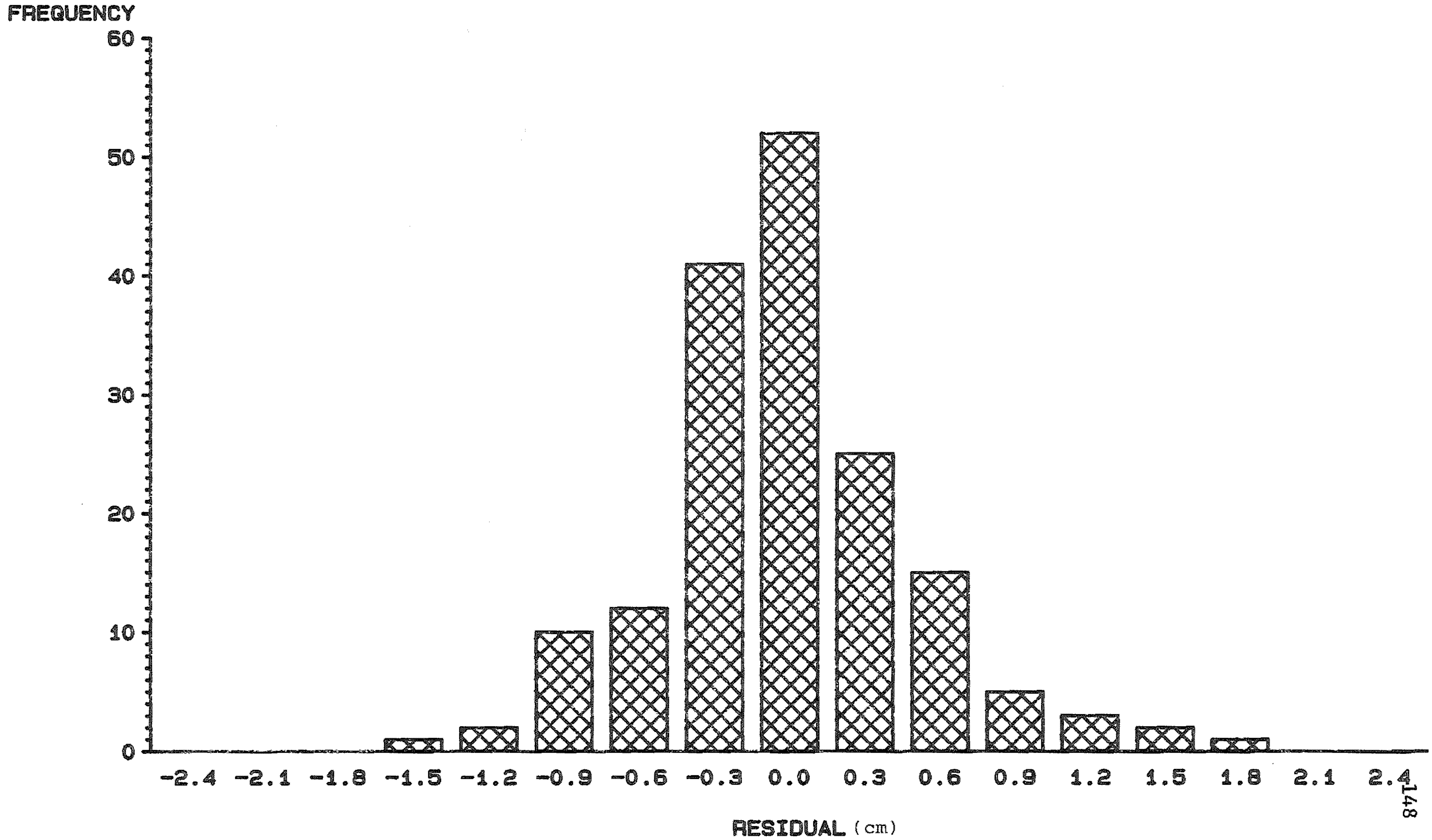


FIG (5.4.2.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF EXTREME STD EQUATION

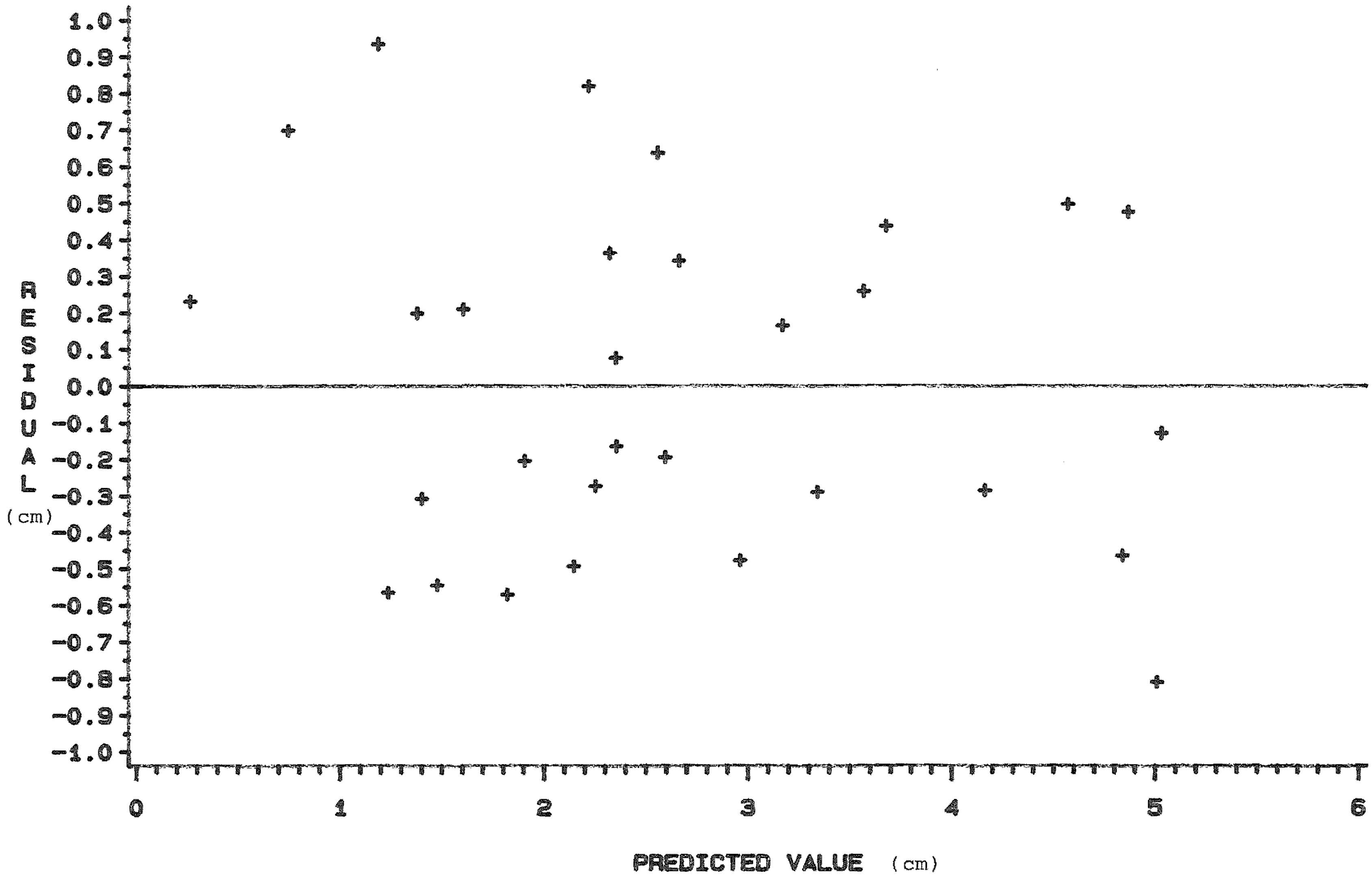


FIG (5.4.2.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF EXTREME STD EQUATION

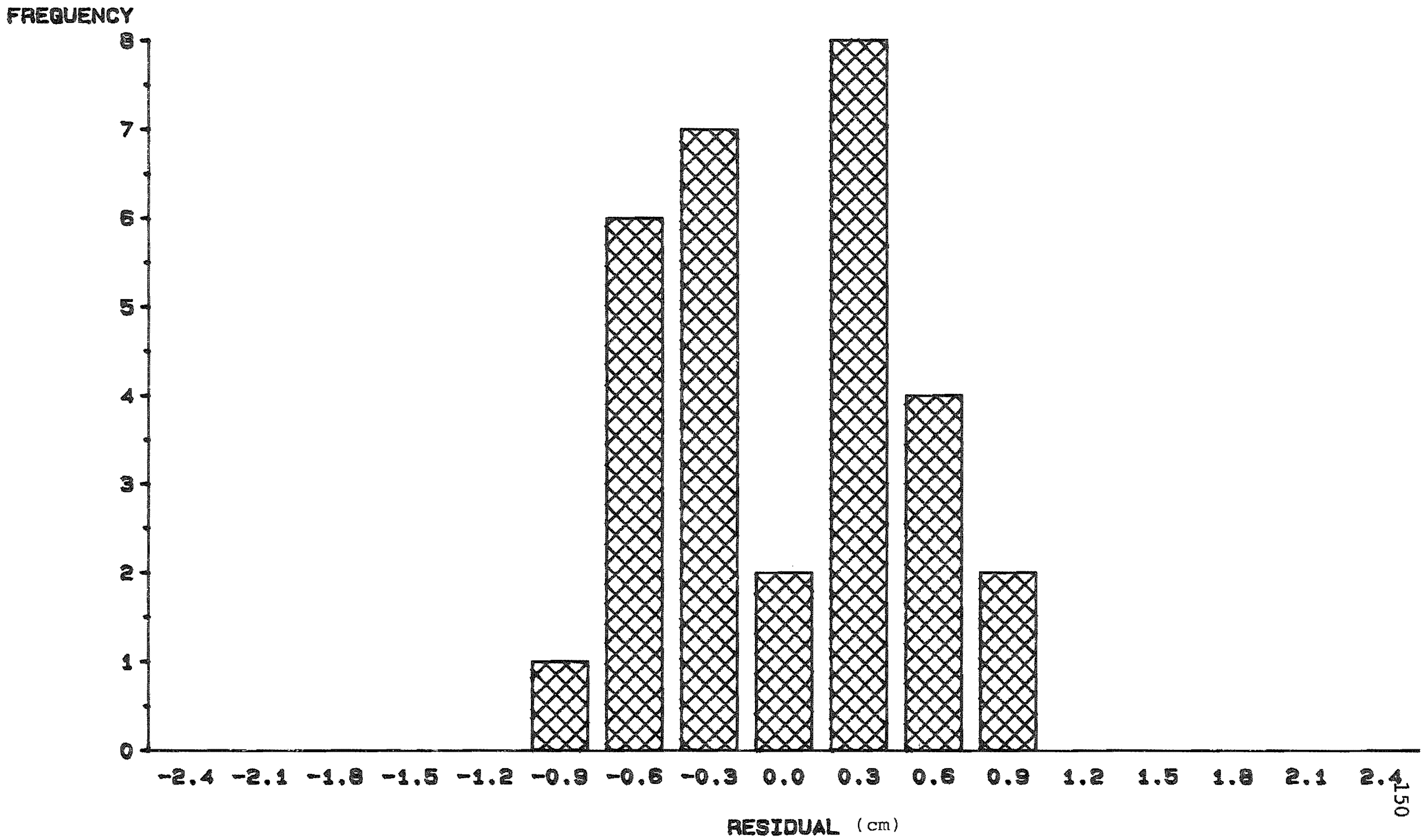


FIG (5.4.2.4): KAINGAROA UNDISEASED STANDS

5.4.3 Projection of the Maximum of the Extreme

Data file for the maximum of extreme projection equations is DMAX.SAS, which has already been described in section 5.4.1. In the file, d_{max1} and d_{max2} were sorted by age and the maximum of maximum diameters of that age was extracted. Equation (5.4.1.1), with d_{xmean2} replaced by d_{xmax2} and d_{xmean1} replaced by d_{xmax1}, was then fitted to those extreme values.

Table (5.4.3.1) shows the estimated values of the parameters of this equation.

Table (5.4.3.1) Estimated statistics of the maximum of the extreme

set	Estimates of α	Estimates of β	STD error of α	STD error of β	n
post 63	1.231708	0.007180	0.0549833	0.000483	168
pre-63	1.537670	0.009289	0.194184	0.001464	30

Fig (5.4.3.1) to (5.4.3.4) are the graphs of residuals and residual bar charts of the maximum of the maximum diameters for the diseased and undiseased stands.

RESIDUAL PLOT OF MAXIMUM EXTREME EQUATION

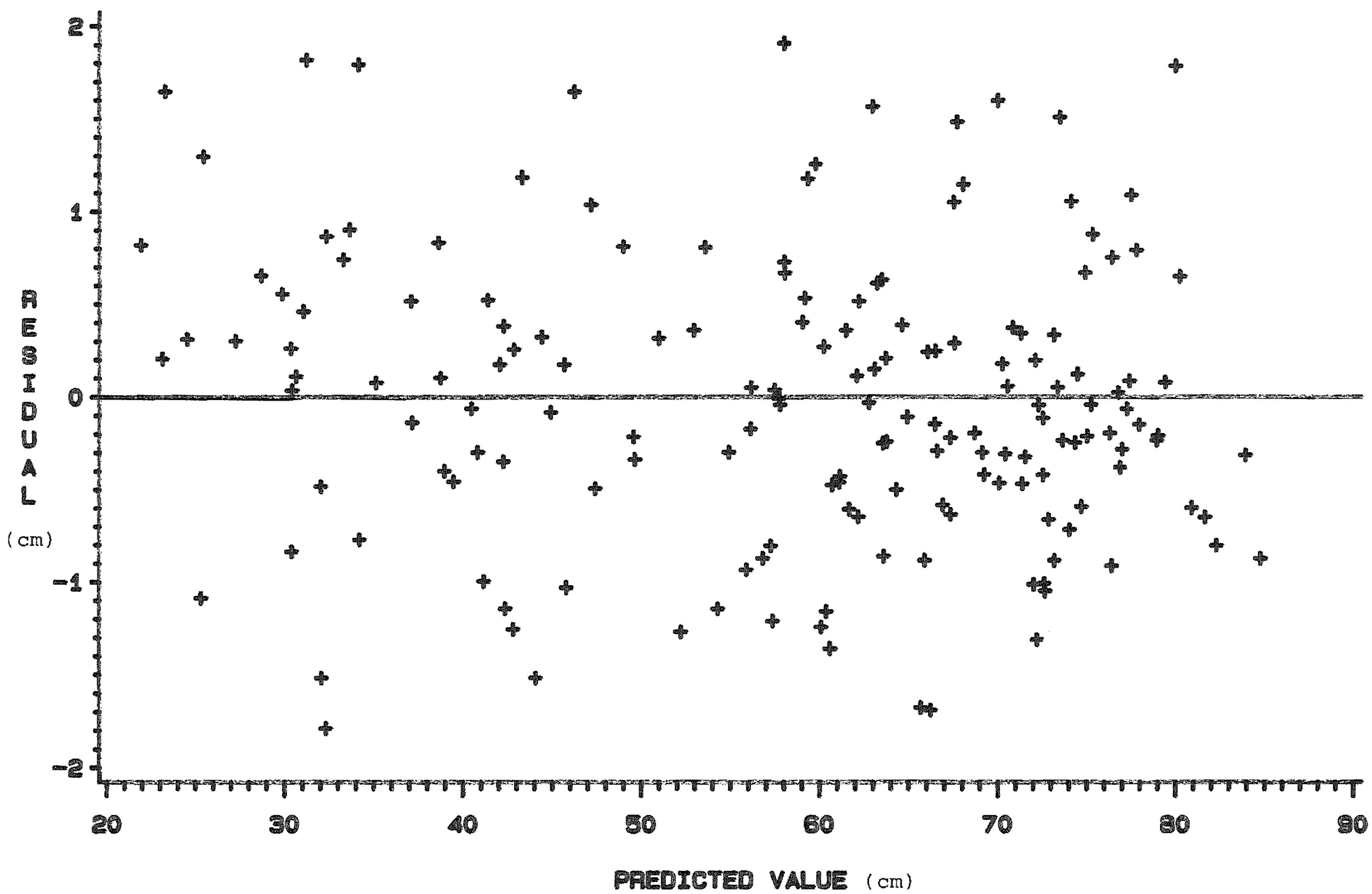


FIG (5.4.3.1): KAINGAROA DISEASED STANDS

RESIDUAL CHART OF MAXIMUM EXTREME EQUATION

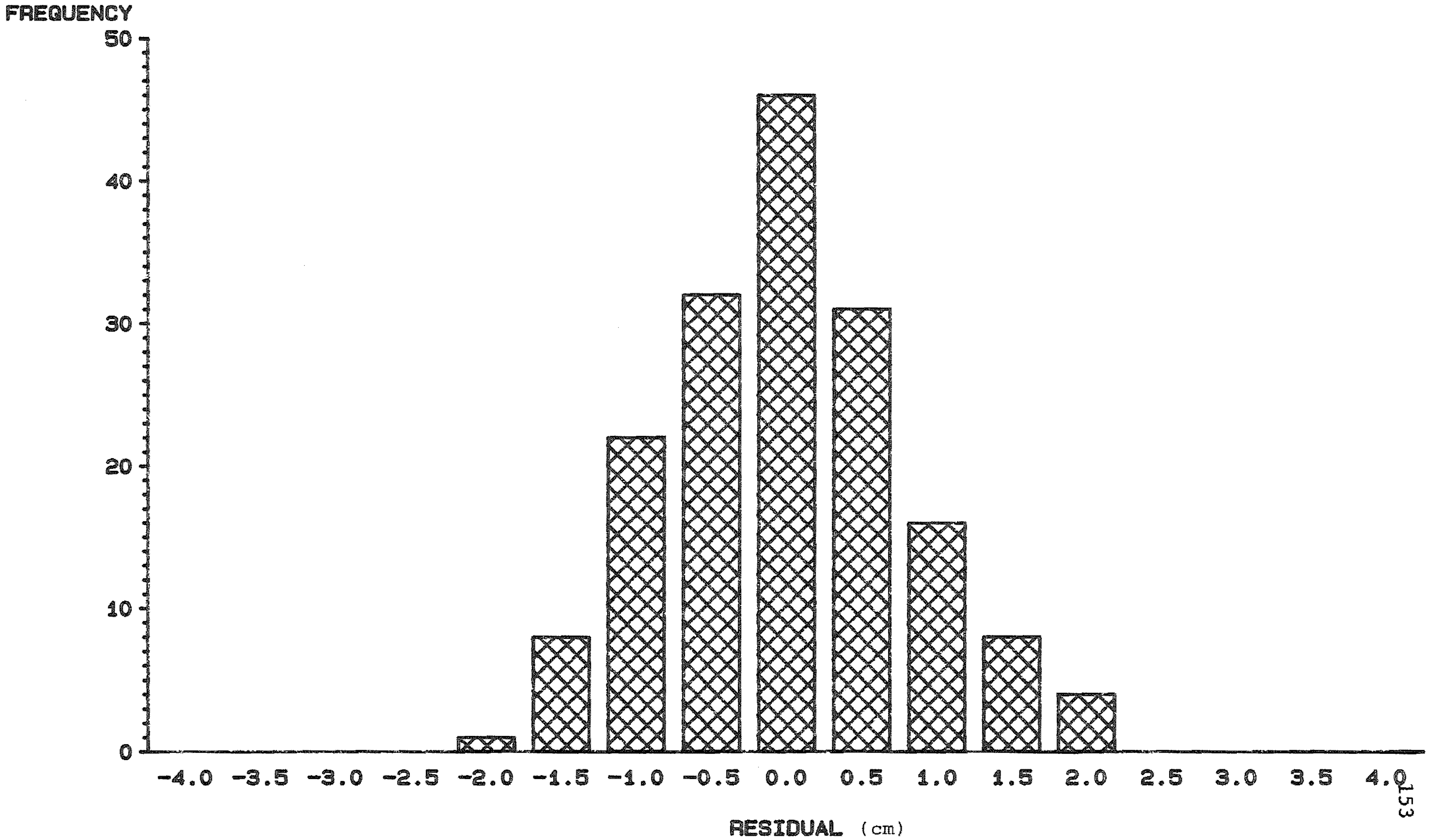


FIG (5.4.3.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF MAXIMUM EXTREME EQUATION

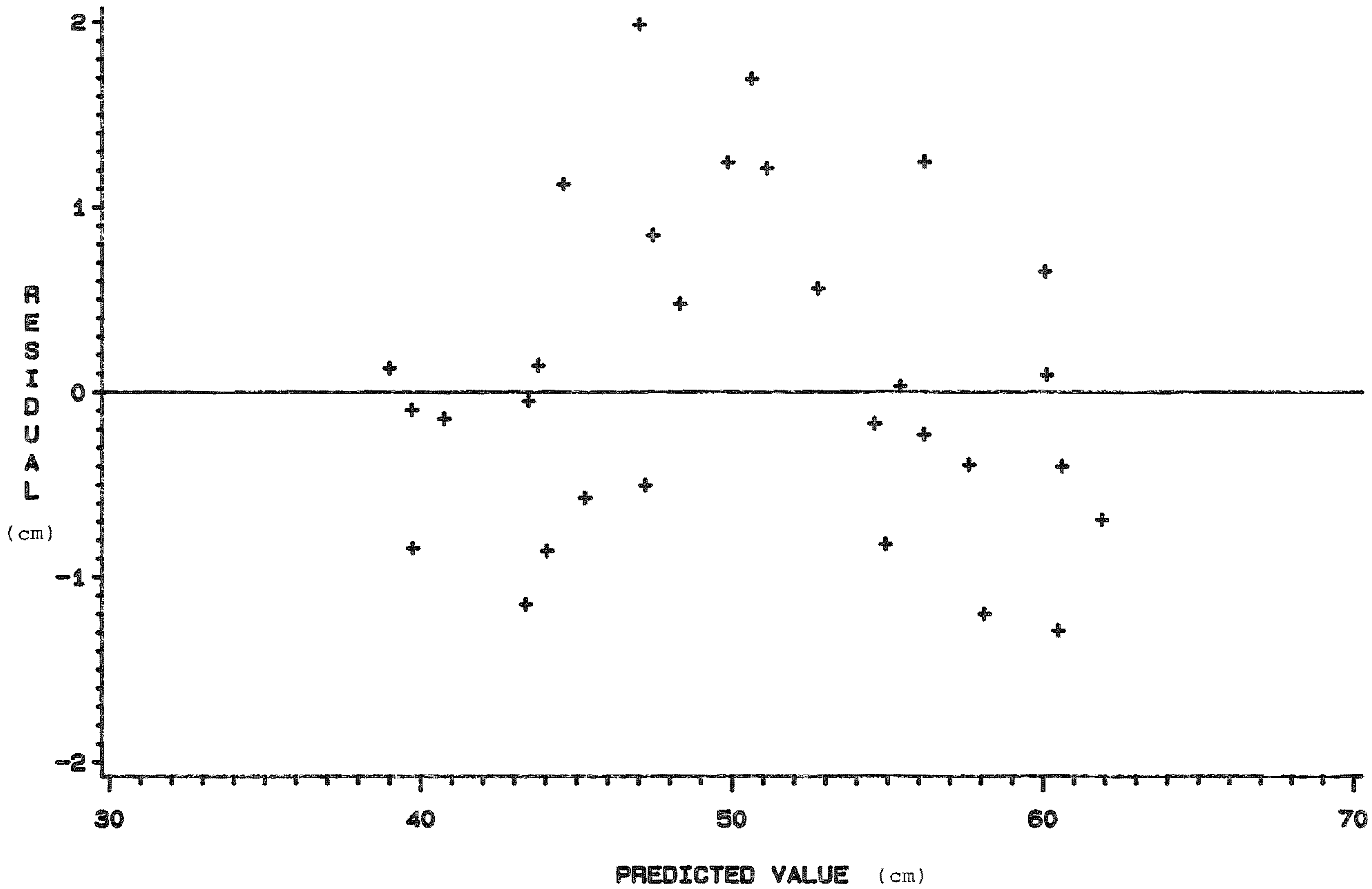


FIG (5.4.3.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF MAXIMUM EXTREME EQUATION

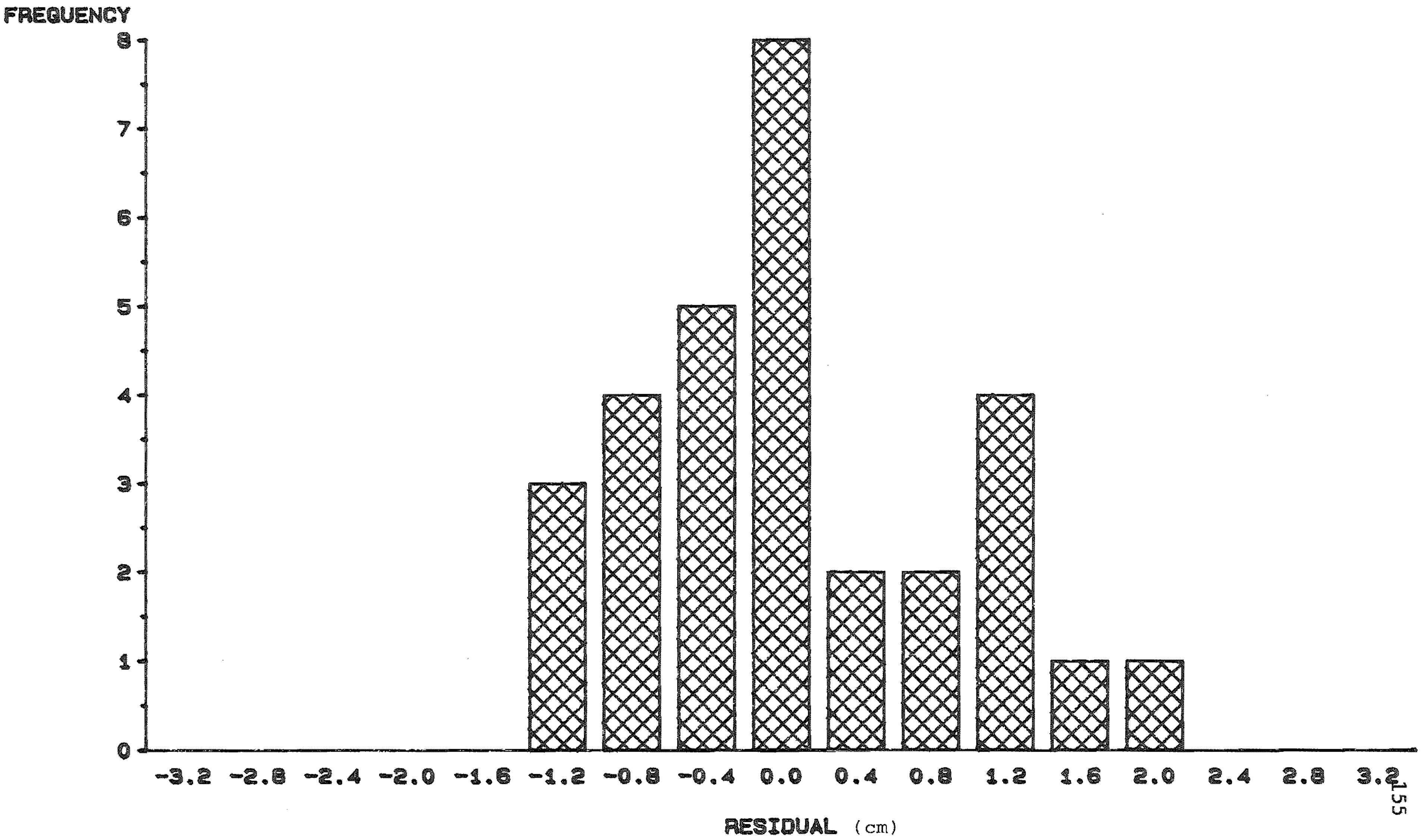


FIG (5.4.3.4): KAINGAROA UNDISEASED STANDS

5.5 THE EXTREME PERCENTILE AND THE RETURN PERIOD

When generally describing continuous univariate distributions, Johnson and Kotz (1970) mentioned the return period. Kuru (1989) cited the return period in his study. He stated "the $100\alpha\%$, $d_{max,\ 1-\alpha}$, which can be regarded as a value that will not be exceeded with an average return period of α-1 units of time is calculated from $X_{1-\alpha} = \epsilon - \theta\log(-\log(1-\alpha))$" but he did not explain what the return period is, did not use a functional form of return period nor did he calculate its value "α-1" (Kuru, 1989. p25).

Based on Gumbel's works (Gumbel, 1941, 1942b, 1945a, 1949b and 1958), the return period has been used in forest diameter distribution modelling in this study. The return period has been defined as rotation age or maximum age of a stand, its functional form presented and interpreted, and its value calculated with respect to Douglas fir plantations in the Central North Island of New Zealand.

Gumbel (Gumbel, 1958; Johnson and Kotz, 1970) used the return period in the studies of floods. He described the return period as follows.

With respect to a dichotomy for a continuous variate, first consider the observations equal to or larger than a certain large value x. Second, consider the observations smaller than this value. Let

$$q = 1-p = F(x) \quad \text{.............................}(5.5.1)$$

be the probability of a value smaller than x. Observations are made at regular intervals of time, and the experiment stopped when the value x has been exceeded once. We ask for the probability w(v) that the excess occurs for the first time at trial v.

The variable v is an integer, limited to the left, but unlimited to the right. If the event happens at trial v, it must have failed in the first v-1 trials, Therefore the probability w(v) is

$$w(v) = pq^{v-1} \qquad (5.5.2)$$

the mean v for which we write T(x) is

$$v = \frac{1}{p} = T(x) = \frac{1}{1-F(x)} > 1 \qquad (5.5.3)$$

the mean T(x) is called the return period.

In terms of floods, the probability that a large flood could happen increases, the greater the number of years of measurements. The return period was interpreted as the mean number of years required for a certain large flood to happen.

In the case of floods, the probability that the largest flood would happen increases with the number of years of measurements, the question is within what time period (return period), T(x), a certain large flood will happen at a specified probability level F(x) (equation 5.5.3). Suppose, when the theory is applied to forestry growth and yield modelling, N PSP samples of size n (n is the number of trees in a plot) in a stand of a certain age are measured yearly. Each year, there will be a maximum diameter, and the probability that the largest diameter would be reached increases with the number of years of measurements. The question now, though, is what is the probability F(x), at which a certain large diameter would appear for a specified time period (return period or age) T(x). If a stand is felled at age T(x), then the age at which the stand reaches its largest diameter is T(x). In other words, the return period T(x) is known; we ask for the probability at which the largest diameter corresponding to that return period occurs. The return period, therefore, is interpreted as an age at which the maximum diameter of the stand has been reached, or the stand's diameter might still be able to grow but is no longer relevant, the stand is felled for instance. Because the return period (or an age), T(x), is known, the probability corresponding to that return period, F(x) (we call it extreme percentile), can be easily calculated:

$$p_x = F(x) = 1 - \frac{1}{T(x)} \quad \text{(5.5.4)}$$

This percentile is used to guide the choice of the percentile on the type III Extreme Value Distribution for calculating the "a" parameter of the Reverse Weibull distribution. Calculating the "a" parameter by combining the type III Extreme Value Distribution with the extreme percentile avoids possible bias that can occur when a percentile is chosen arbitrarily

Some conditions about the assumption of using the return period are not strictly obeyed here. Firstly, it is assumed that the population from which the sample will be drawn contains all information needed in the experiment, including a range of ages. The measurements are taken instantly and continuously and the number drawn is increased to cover a range of ages. In practice, PSP measurements in New Zealand are taken yearly or 2 yearly over a period of time, This has not departed too much from the original assumption. Secondly, PSP measurements are systematically correlated. Although there are concerns about this (e.g. Sullivan et al., 1976), most forestry research, including this study, still has to be based on this sort of repeatedly measured data. But this is unlikely to cause serious problems in practice if the sample is large (Clutter et al., 1983).

For the Douglas fir data used in this study, the largest age in the data set is 82. So the percentile at which the largest diameter is located is: $p_x = F(x) = 1-1 / T(x) = 1-1 / 82 = 0.9878$ or 0.99. If a rotation age of 40 is used rather than the largest age in the data set, then the percentile is $F(x) = 0.975$ or 0.98. For some species such as radiata pine, where the rotation age might be as low as 25 years, the percentile is $p_x = 1-1 / 25 = 0.96$.

It is believed that for any species the percentile to be chosen should not be less than 95 percent. Otherwise the largest diameter chosen in this manner is likely to be exceeded during the specified period of years.

The extreme percentile is independent of any distribution; it is easy to calculate and use.

5.6 A SUMMARY OF THE PROPOSED PROCEDURE AND THE FIT OF THE DIAMETER DISTRIBUTION MODEL

5.6.1 Type III Extreme Value Distribution and Derivation of Reverse Weibull Location Parameter a

As demonstrated in section 5.3.2, the extreme value distribution to be used is the type III, the Reverse Weibull Distribution, when the initial distribution assumes the Reverse Weibull. Thus the type III Extreme Value Distribution can be given by rewriting the Reverse Weibull distribution function with its corresponding parameters denoted by a subscript x. The cumulative distribution function of the type III Extreme Value Distribution is

$$f(x) = \exp\left[\left(-\frac{a_x - x}{b_x}\right)^{c_x}\right] \qquad -\infty \le x \le a_x \ldots\ldots\ldots\ldots (5.6.1.1a)$$

$$= 1 \qquad x \ge a_x \ldots\ldots\ldots\ldots\ldots (5.6.1.1b)$$

with density function

$$f(x) = \frac{c_x}{b_x}\left[\left(\frac{a_x - x}{b_x}\right)^{c_x - 1}\right] \exp\left[-\left(\frac{a_x - x}{b_x}\right)^{c_x}\right] \ldots\ldots\ldots\ldots (5.6.1.2)$$

In (5.6.1.1a), f(x) is a percentile on the point x (of the cumulative distribution) below which all x values are smaller. Thus

$$p_x = f(x) = \exp[(-\frac{a_x - x}{b_x})^{c_x}] \qquad (5.6.1.3)$$

When the percentile p_x equals the maximum value chosen (in this case, we will choose a value determined by the return period), then, x is the maximum diameter of the stand by definition. Obviously, x = a, the location of the initial distribution. Solving (5.6.1.3) for x, or the location parameter of the Reverse Weibull distribution, we have

$$a = x = a_x - b_x[-\ln(p_x)]^{1/cx} \qquad (5.6.1.3)$$

The parameters of the type III extreme value distribution, a_x, b_x and c_x and the extreme percentile, p_x, have been determined in section 5.4.1 to 5.5; thus equation (5.6.1.3) is ready to be solved for the location of the initial distribution.

5.6.2 A Summary of the Proposed Procedure for Deriving the Diameter Distribution

In terms of the diseased stand, the standard deviation, s, of a stand is projected by equation (5.2.3.1). Then, values projected by equations (4.4.3.7) and (4.4.6.4) are used to solve equation (5.2.1.5) to get the quadratic mean diameter d_g. The arithmetic mean diameter, d, was obtained through equation (5.2.1.7) by substitution of s and d_g obtained earlier. With values of s and d (in place of $\bar{x}$) calculated, the Reverse Weibull parameters c and b are ready to be solved using equations (5.2.1.3) and (5.2.1.4).

The Reverse Weibull parameter a is estimated by the type III extreme value distribution using an extreme percentile.

The maximum of the largest diameter, D_{xmax}, is projected by equation (5.4.1.1) with coefficients given in table (5.4.3.1). The mean, D_{xmean}, and the standard deviation, D_{xstd}, of the largest diameters are projected using the same equation form (5.4.1.1) but with different coefficients (tables 5.4.1.1 and 5.4.2.1 respectively). With D_{xstd} in place of s and D_{xmean} in place of $\bar{x}$, equations (5.2.1.3) and (5.2.1.4) are ready to be solved for the shape parameter, c_x, and scale parameter, b_x, of the Type III Extreme Value Distribution. The location parameter, a_x, of the type III is set equal to the value projected by equation (5.4.1.1). Then, the Reverse Weibull parameter a is calculated by equation (5.6.1.3) with a_x, b_x, c_x just estimated and percentile value obtained from equation (5.5.4). At this point, all the parameters of the Reverse Weibull have been estimated and a complete Reverse Weibull distribution is established.

The usual procedure is employed to work out the stand table: probability assigned to each dbhob class times the stems/ha projected by equation (4.4.6.4) gives numbers of trees in each class. Volume of individual tree is calculated from equation (5.8.1), using dbhob and height projected by equation (5.7.1). Tree volume times numbers of trees in each dbhob class gives the volume/ha in each class. This procedure is available at any point in time and projection of future stand structure can be made by repeating this calculation for the ages desired.

This procedure can also be used in conjunction with the parameter recovery procedure. To do so, parameter a of the Reverse Weibull can be estimated by the procedure just described and parameters c and b can be recovered from stand statistics to keep the compatibility between estimations made by stand level projection equations and diameter distributions.

5.6.3 The Fit of the Diameter Distribution Models

Normally, diameters of all tree in a plot are measured when the plot is visited each time. To check the fit of the diameter distribution model by residual plotting, a file of dbhob classes was formed. The diameter measurements were sorted by 2 cm classes for all plot measurements involved in this study. All those sorted values form a file of dbhob classes, which are the actual values of the diameter distribution. By using the equation systems developed, a predicted diameter distribution can be calculated for each corresponding age used to form the actual diameter distribution. For a given age, a residual of diameter distribution (actual value - predicted value) can be calculated. Those residuals are then plotted to detect possible bias. Fig (5.6.1) is a graph of residuals corresponding to the actual and predicted diameter distributions for diseased, thinned stands. It shows that the diameter distribution model gives an unbiased prediction of the diameter distribution of Douglas fir crops in the Central North Island of New Zealand.

Fig (5.6.2) is the projected diameter distribution of the same population and same equations system, except that the extreme value distribution is the type I. It can be seen that the type III extreme value distribution resulted in a much better fit, thus empirically confirming that the type three is theoretically the correct choice.

The model is also tested by the Kolmogorov-Smirnov test and the result is consistent with the visually assessed patterns.

RESIDUAL PLOT OF DIAMETER DISTRIBUTION

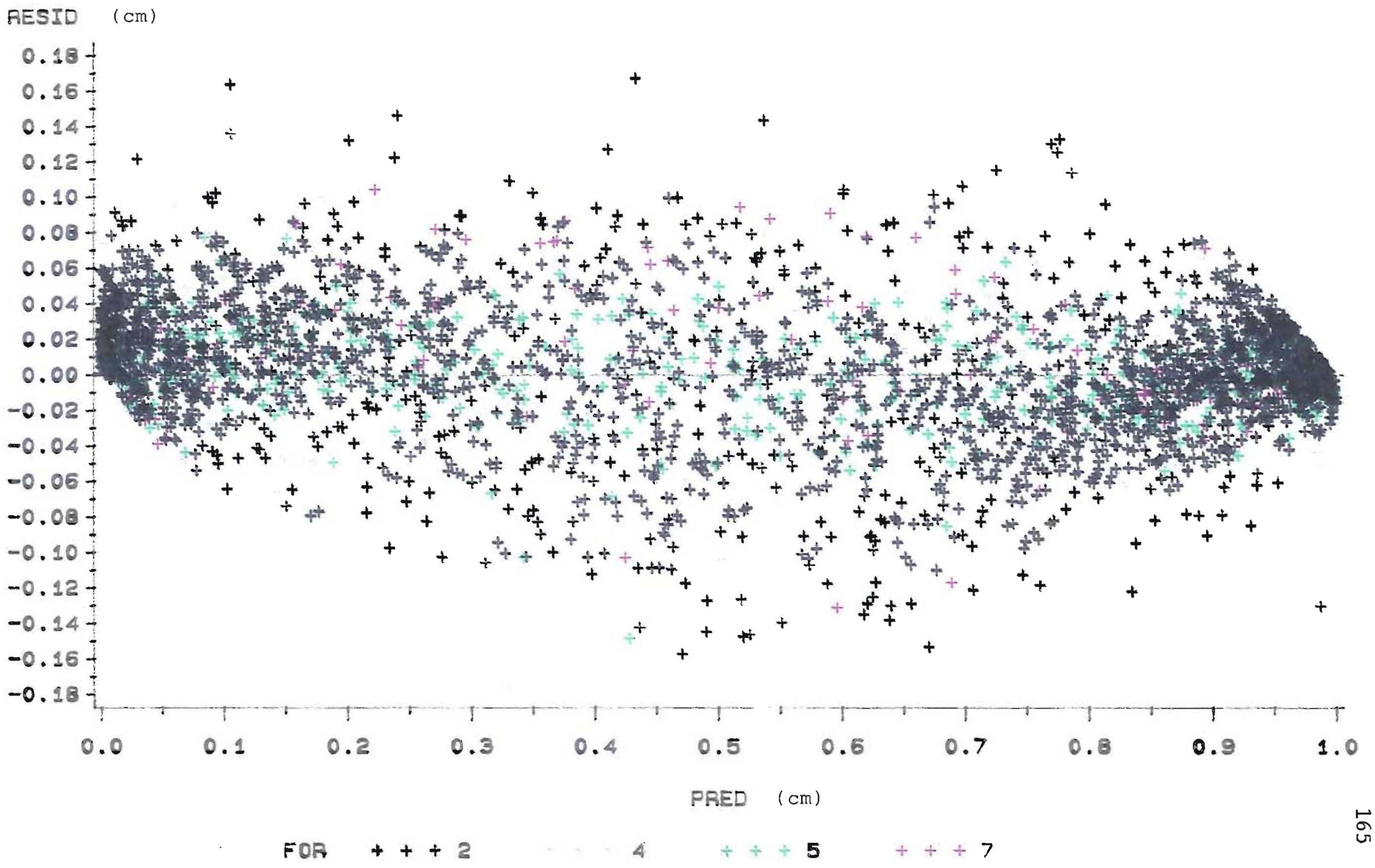


FIG (5.6.1): USING TYPE III EXTREME DISTRIBUTION

RESIDUAL PLOT OF DIAMETER DISTRIBUTION

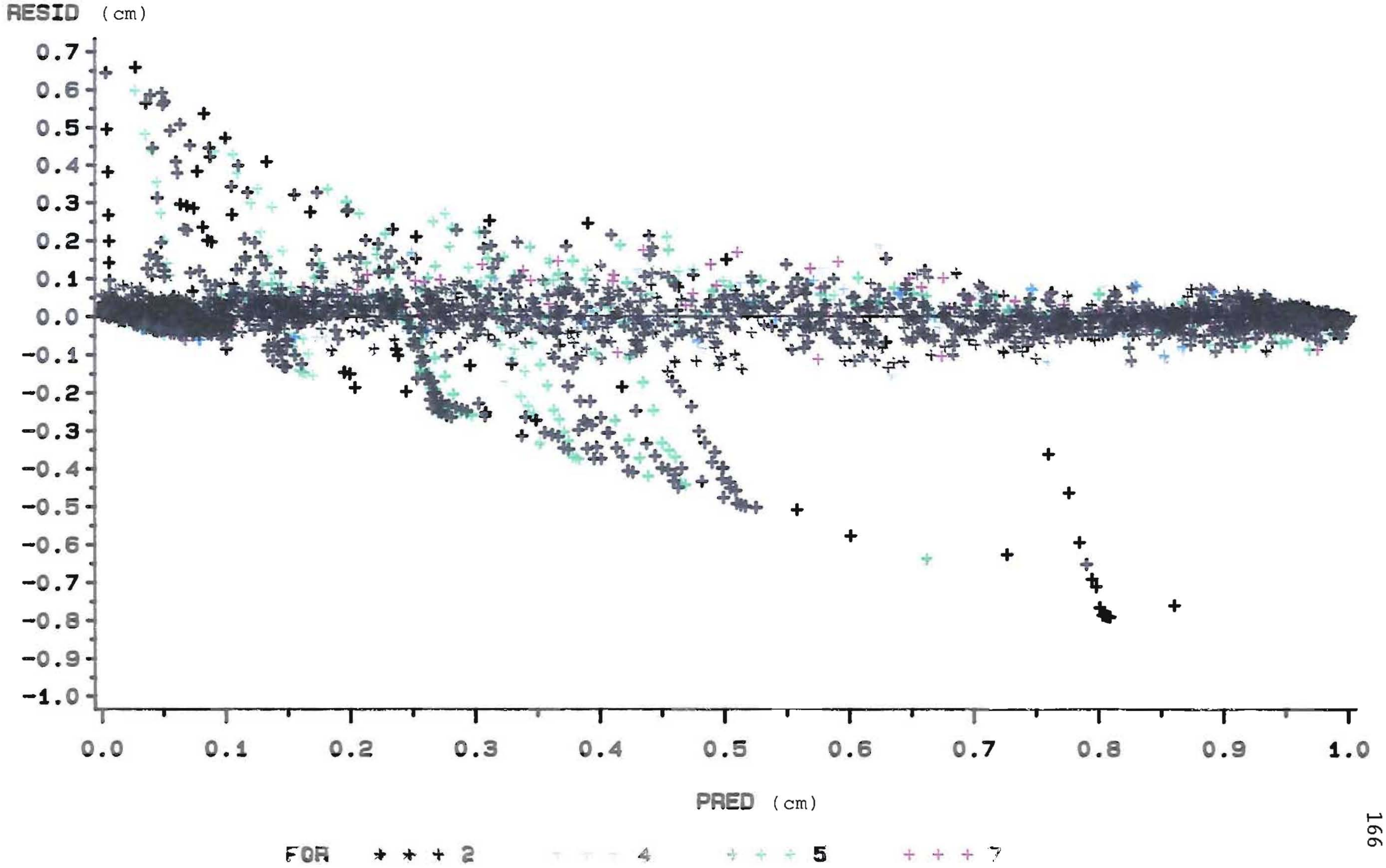


FIG (5.6.2): USING TYPE I EXTREME DISTRIBUTION

5.7 HEIGHT CORRESPONDING TO MID-POINT DIAMETER EQUATION

The equation is:

$$h = \alpha + \beta dbh + \gamma h_{100} \quad \text{..........................} (5.7.1)$$

The variable h_{100} is introduced into the equation to reflect site variations among the forests of the region to which the model applies. The equation produced a good fit, though variations are larger than for the mean top height equation given in chapter 4.

Table (5.7.1) shows the estimated statistics of this equation. Fig. (5.7.1) to (5.7.4) are the residual patterns and residual charts of this equation for diseased and healthy stands.

Table (5.7.1) Estimated statistics of height corresponding to mid-point diameter

set	Estimates and STD error of			n
	α	β	γ	
post-63	-1.360129929 0.0881154649	0.736864222 0.005875345	0.199586622 0.003865749	2352
pre-63	-1.600022307 0.5060354292	0.742321007 0.021919004	0.209535766 0.012145586	220

RESIDUAL PLOT OF HEIGHT EQUATION

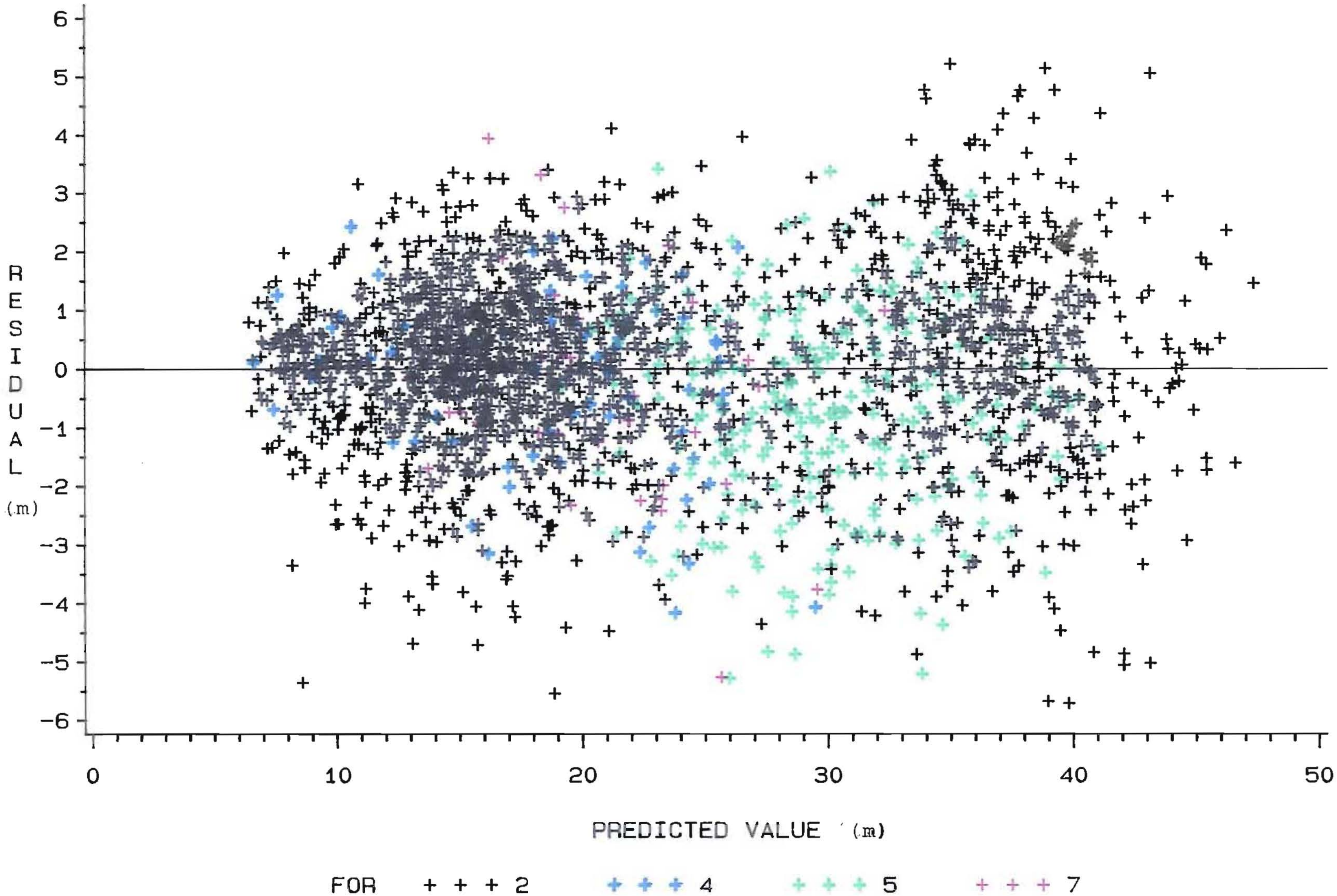


FIG (5.7.1): KAINGAROA DISEASED STANDS

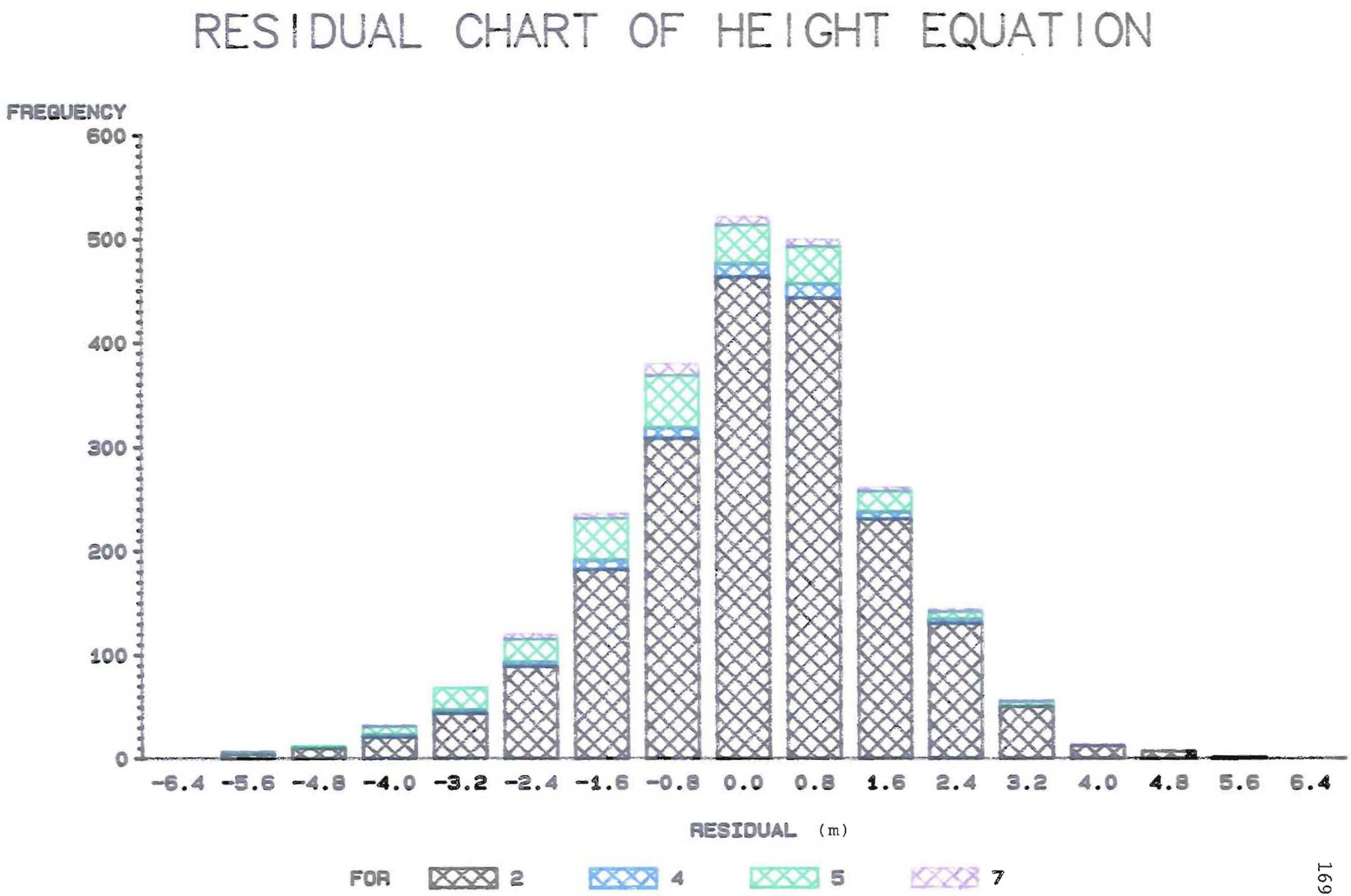


FIG (5.7.2): KAINGAROA DISEASED STANDS

RESIDUAL PLOT OF HEIGHT EQUATION

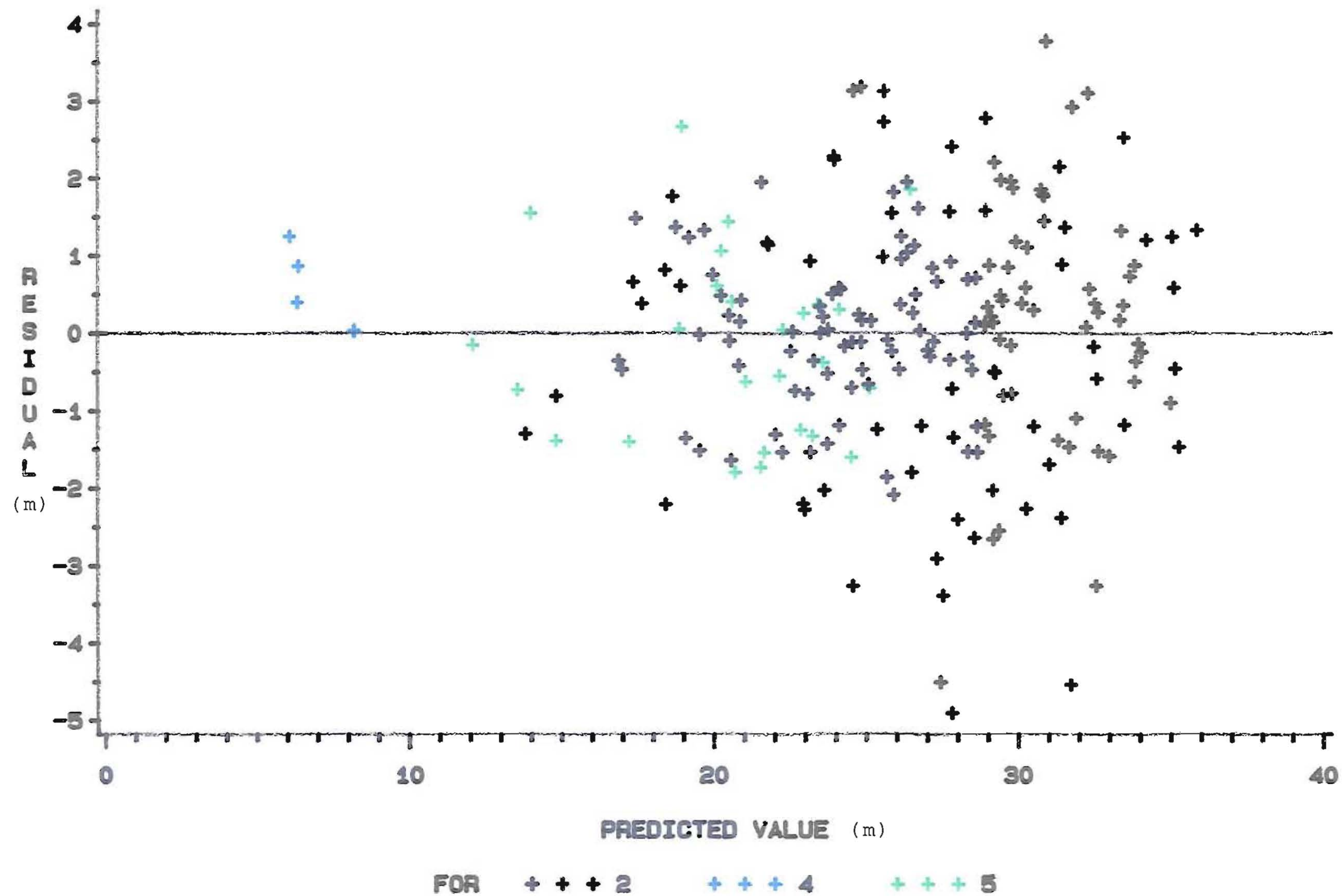


FIG (5.7.3): KAINGAROA UNDISEASED STANDS

RESIDUAL CHART OF HEIGHT EQUATION

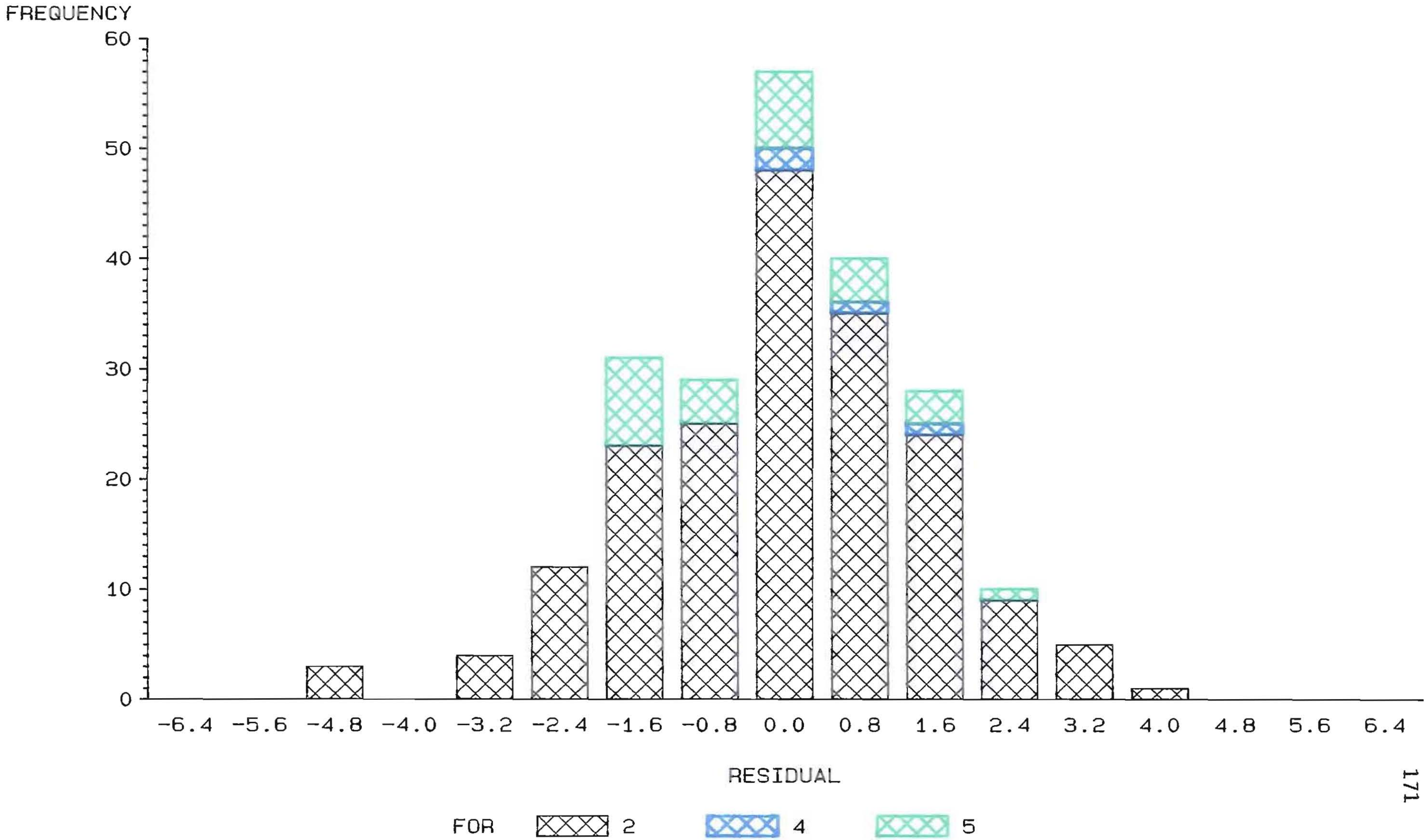


FIG (5.7.4): KAINGAROA UNDISEASED STANDS

5.8 DERIVATION OF TOTAL STEM AND MERCHANTABLE VOLUMES

The FRI PSP system uses regional, two-dimensional, individual tree volume equations to allow derivation of plot total stem volume. It does not produce the individual tree merchantable volumes. The most appropriate total stem tree volume equation for the species and a corresponding compatible taper equation have to be used. The equations are, respectively

$$\ln(v) = b_1\ln(d) + b_2\ln(h^2/(h-1.4) + b_o \text{.......}(5.8.1)$$

$$d_k^2 = v/(Kh)(b_3x + b_4x^2 + b_5x^3 + b_6x^4 + b_7x^5 + b_8x^{b9})\text{.........................}(5.8.2)$$

$$v_m = v/(b_{10}x^2 + b_{11}x^3 + b_{12}x^4 + b_{13}x^5 + b_{14}x^6 + b_{15}x^{b9+1})\text{......................}(5.8.3)$$

where

$\ln(v)$ = natural logarithm of total volume inside bark;

d_k = diameter over bark at breast height;

b_o, b_1, b_2 = coefficients. and in the taper equation:

h = total tree height (m);

x = (h-height to the diameter d_k)/h (m);

v_m = volume from tip to height at diameter d_k (m^3/ha).

The rest of the symbols in the taper equation are all coefficients with values shown in appendix 5.

As described earlier, the volume generated by these equations can be made compatible to that summed up from dbhob classes by using the parameter recovery procedure (see, for example, Cao *et al*., 1982).

5.9 RELIABILITY OF THE MODEL COMPONENTS

Maximum predicted errors for the maximum diameter projection equations are within 2 cm for both diseased and undiseased stands, while the mean predicted error will not exceed 0.7 cm. For the standard deviation projection equation, maximum predicted errors are within 0.7 cm for both diseased and undiseased stands while mean predicted error are within 0.1 cm. Although observations available for fitting the extreme value parameter projection equations are few, the resultant fits are reasonably good; all mean residuals lie within 0.7 cm and will not exceed 2 cm in any cases for the maximum of extreme. Predicted error for the mean of extreme is also around about 2 cm and average residuals lie within 0.5 cm. All the equations produced a reasonably good fit except the equation for standard deviation of the extreme, in which the parameter β is not significant, perhaps due to a small numbers of observations for fitting the equation. The model appears, therefore, to produce precise prediction of the diameter statistics of Douglas fir plantations in the study region.

CHAPTER 6 VERIFICATIONS AND EVALUATIONS

6.1 VERIFICATIONS

The soundness and reliability of the prediction equations were judged partly from visual inspection of graphical plotting of residuals against predicted values and other variables, from residual bar charts and from estimated asymptotic standard error. The residual bar chart is a somewhat subjective approach, however, and so other statistics were used to check the goodness of fit more objectively. Those statistics are the mean, the maximum (max), the minimum (min) and the standard deviation of the residuals (STD) as set out in the following table (Table 6.1.1). Projection precision was also assessed on the size of the absolute mean residual (ABSMR), which is also presented in the table, together with the two extremes, the maximum and the minimum residuals. In the table, the symbols Min, Mean, Max, EQN and n refer to the minimum, mean and maximum residual of an equation (EQN) fitted to a data set of n observations;

ABSMR = absolute mean residual of an equation;

G = basal area projection. The subscripts d and u stand for diseased and undiseased stands respectively and will be the same for all equations in the table.

Table (6.1.1) Residual statistics of the prediction models

EQN for	Min	Mean	Max	ABSMR	STD	n
site index	-1.9	0.00874	2.1	0.42	0.5713	1949
G_d	-2.37	0.02563	2.5	0.53	0.6993	1746
G_u	-2.27	0.08111	2.9	0.83	1.0197	157
V_d	-43.01	-0.01136	41.50	6.74	10.2352	1639
V_u	-33.79	-0.11557	28.61	7.12	9.8282	163
V_{md}	-3.98	-0.02347	3.99	0.71	0.9954	1653
V_{mu}	-2.96	-0.01871	3.67	0.61	0.9204	157
Mort-ality	-117	-2.76359	121	16	27	790
d_{maxd}	-.2.34	0.00611	2.32	0.62	0.61577	1662
d_{maxu}	-3.20	-0.00120	3.40	0.83	1.04882	116
d_{stdd}	-0.70	0.00590	0.61	0.10	0.14458	1451
d_{stdu}	-0.63	0.01144	0.61	0.17		114
d_{xmaxd}	-1.79	-0.01246	1.91	0.62	0.78669	168
d_{xmaxu}	-1.29	0.06836	1.99	0.70	0.87648	30
d_{xstdd}	-1.47	-0.01485	1.74	0.36	0.49627	169
d_{xstdu}	-0.68	0.03403	0.91	0.42	0.48148	30

Table (6.1.1) continued

EQN	Min	Mean	Max	ABSMR	STD	n
d_{xmeand}	-1.85	0.01776	2.02	0.43	0.60376	170
d_{xmeanu}	-1.38	0.07114	1.08	0.44	0.57016	30
h_d	-5.71	-0.00000	5.21	1.22		2352
h_u	-4.90	0.00000	3.78	1.14		220

V = total volume/ha;

V_m = merchantable volume/ha;

M = mortality;

d_{max} = maximum diameter projection;

d_{std} = standard deviation projection;

d_{xmax} = maximum of maximum diameters;

d_{xstd} = standard deviation of maximum of maximum diameters;

d_{xmean} = mean of maximum of maximum diameters;

h = height corresponding to mid-point diameter.

Theoretically, the mean residual should be equal to zero. In reality, estimated mean residual for an equation is rarely zero exactly. It should, however, be close to it. The value of the mean residual is more important than its sign. Any prediction made from an equation fitted with least-squares is an estimation which may over- or under-predict the true values. The issue is how great a departure should be tolerated. The absolute mean residual, calculated by summing up all absolute residuals divided by the sample size, indicates the average distance of this departure. For example, the site index equation in Table (6.1.1) has a mean residual of 0.00874 meaning a slight under-prediction, but the ABSMR is 0.42, implying an average departure of 0.42 m. When measuring heights in the field, errors resulting from such sources as inaccuracy in equipment, visual error, weather at the time of measurement etc. would produce an error of at least about 0.2 m. Thus the site index equation can give good predictions. Because this site index equation is developed using the difference equation method, unlike the site index equation developed by Burkhart and Tennent (1977) as shown in Fig. 4.4.2.1 to 4.4.2.3, a specific choice of an index age is made simply for labelling purposes and has no effect on the shape of the site index curves produced. The equation is also easier to use. In this equation, site index is directly a function of mean top height and age thus does not require an iterative procedure for the solution. The equation has wider application in relation to Douglas fir plantations in the Central North Island. It applies to Kaingaroa, Pureora, Whirinaki and Waimihia while Burkhart's model applies to Kaingaroa only. Apart from these, it also has produced a good fit that is almost identical to that developed by Burkhart and Tennent (Fig 4.4.2.1 to 4.4.2.6).

In terms of the basal area projection equation, bias is larger for the disease-free than for the diseased stands. This arises mainly because of the smaller number of observations in the disease-free group. Nevertheless, maximum predicted error will not exceed 3 m^2/ha. On average predicted error will be 0.53 m^2/ha for the diseased stands and 0.83 m^2/ha for healthy stands. Thus the equations will produce precise predictions of future basal area/ha although the pattern of residuals shows some bias (4.4.3.1).

The standard error of the intercept for the total volume equation in the post-1963 group is nearly one third of the estimate but residuals distributed much nicely when the intercept is retained. The signs of the mean residuals for the volume equations are negative. This means there will be an over-estimation of volume. But on average the estimated error will be within 8 m^3/ha and will not exceed 43 m^3/ha in extreme cases.

In terms of error mean squares, residual pattern, bias and goodness-of-fit, the merchantable volume equation is one of the best-fitted equations to be found for this data set. It was noted that extreme residual values for merchantable volume lie within 4 m^3/ha, and the mean residual value is 0.71 m^3/ha. These low values arise because the merchantable volume is estimated from total volume. Actual residual values of the merchantable volume equation are really additional to the residual values for the total volume equation.

The mean residual value for the mortality equation is -3 trees/ha, which implies an over-prediction of stocking. Mean prediction error is 16 trees/ha. In extreme cases, predicted error can be up to 121 trees/ha.

There is no untoward bias for the major equations in the model, namely mean top height, net basal area/ha, net total stem volume/ha, merchantable volume/ha and mortality except in the cases where the numbers of observations available for fitting the equations are small. For example, equations for estimating basal area/ha or number of trees/ha after thinning is slightly biased due to an insufficient number of measurements available for fitting the equations. These inaccuracies can be avoided if the user specifies both the basal area/ha and number of stems/ha after thinning.

6.2 VALIDATION

Models are best validated by an independent data set. However, such data sets are not available in most situations, as is the case here for Douglas Fir Central North Island Growth Models version 1 and 2. The overall data set was large but 75% of the measurements refer to Kaingaroa forest and the other six forests shared the remaining 25%. Setting aside a validating data set from within the whole data set, therefore, would have resulted in too small a number of measurements for Pureora, Waimihia, Whirinaki forests, and for disease-free stands to represent growth reliably. The reliability of the models themselves, consequently, would have suffered adversely if some had been retained for validation only. The whole data set then was used for the construction and evaluation as previously described.

Empirical testing has been carried out in routine practice, however. The models have been in use by the New Zealand Forestry Corporation and other organisations since their completion. Users have reported finding them able to give good prediction for all stand statistics except stem mortality in DFCNIGM1, which appeared to be over-predicted. A revised stand mortality equation in DFCNIGM2 appears to have overcome this deficiency.

The models have also been tested in an independent study of South Island Douglas fir by Law (1990). In her study, three management regimes were specified:

1. initial stocking 1600 stems/ha, unthinned;
2. initial stocking 1600 stems/ha, thinned at 10 m to 400 stems/ha;
3. initial stocking 1600 stems/ha, thinned at 10 m to 600 stems/ha.

Below is the table she presented, in which outputs for the three regimes from DFCNIGM2 were compared with that from SIDFR, a model she developed for South Island Douglas fir plantations.

It was found that DFCNIGM2 can give predictions similar to that by SIDFIR except for the first regime, where stocking predictions from SIDFIR are greater than DFCNIGM (-) (no disease) but less than DFCNIGM (+) (disease present); and predicted basal area/ha from SIDFIR is 20% higher than DFCNIGM (+), where DFCNIGM refers to DFCNIGM version 2 model.

DFCNIGM3 has not yet been validated through users or against an independent data set, but its stand predictions are constrained by the validated stand model components of DFCNIGM2, around which the diameter distribution model has been derived.

Table (6.1.1) Comparison of SIDFIR and DFCNIGM2

regime/model		height m	stoc-king /ha	basal area m^2/ha	mean dbh cm	volume m^3/ha
1	SIDFIR	33.3	1023	93.5	34.1	1117
	DFCNIGM(-)	32.5	978	103.4	36.7	1208
	DFCNIGM(+)	32.5	1176	75.1	28.5	868
2	SIDFIR	33.3	354	72.0	50.9	895
	DFCNIGM(-)	32.5	371	91.7	56.1	1073
	DFCNIGM(+)	32.5	379	70.2	48.6	816
3	SDIFIR	33.3	243	55.2	53.9	659
	DFCNIGM(-)	32.5	237	78.7	65.0	921
	DFCNIGM(+)	32.5	242	56.4	54.5	656

6.3 PROJECTION LOGIC IN TERMS OF COMMON BIOLOGICAL RELATIONSHIPS

For a given site index, starting basal area/ha and initial stocking (e.g. 31 m, 45 m^2/ha and 1000 trees/ha respectively) predicted volume increased with age (Table

6.3.1). For a given age, initial basal area/ha and stocking level (e.g. 20 years, 45 m^2/ha and 1000 trees/ha respectively), volume forecasts increased as site index increased (Table 6.3.2). For a given age, site index and stocking (e.g. 20 years, 30 m and 1000 trees/ha respectively) volume increased as basal area increased. [Table (6.3.3)]. These demonstrate that projection is biologically realistic.

Table (6.3.1) For a given site index, G/ha and N/ha, volume increased with age

Age (yrs)	Top height (m)	total volume (m^3/ha)	merch. volume (m^3/ha)
20	17.0	290	213
25	20.8	347	255
30	24.6	405	297
35	28.0	456	335
40	31.0	502	369
45	33.6	542	398
50	35.9	576	423
55	37.9	606	445
60	39.7	632	465

Table (6.3.2) For a given age, G/ha and N/ha, volume increased as site index increased

site index (m)	volume (m^3/ha)	merch. volume (m^3/ha)
20	194	143
25	235	173
30	275	202
35	316	232
40	356	262
45	397	292
50	438	321

Table (6.3.3) For a given age, site and G/ha volume increased as basal area/ha increased

Basal area (m^2/ha)	Volume (m^3/ha)	Merch. volume (m^3/ha)
30	184	103
35	215	136
40	245	169
45	275	202
50	306	235
55	336	268
60	366	300

6.4 LIMITATIONS TO APPLICABILITY OF THE MODELS

(1) The ages in the data set occurred largely between 15 and 70 years. The models should be applied to stands of age outside those limits only with extreme caution, particularly at the lower end of the range.

(2) The models apply strictly to forests of the Kaingaroa group, i.e. Kaingaroa, Pureora, Waimihia and Whirinaki. There are reservations about their suitability for Pureora because of the small numbers of measurements from this forest.

CHAPTER 7 DISCUSSION OF RESULTS

7.0 CAPABILITY, SENSITIVITY AND PRECISION OF THE MODELS

This study has produced three growth and yield models for forecasting production from Douglas fir in the Central North Island. DFCNIGM1 and DFCNIGM2 are stand level models that have equations for predicting mean top height or site index, net basal area per hectare, stocking and both total stem and merchantable volumes/ha. DFCNIGM3 is a diameter distribution model that is compatible with DFCNIGM2 and contains additional equations for projecting maximum dbhob, mean dbhob and standard deviation of dbhob so that stand and stock tables can be generated. All the equations show a precision that is as good as or better than similar existing growth and yield models. Thus they allow managers to produce reliable yield forecasts.

Silviculture practices in Douglas fir plantations have been different to those conducted in Radiata pine. In Douglas fir crops, there has been little or no pruning while thinnings have been delayed to promote smaller branches and closer annual rings needed to meet desired timber qualities. Because thinnings have been carried out later in life, they are mostly production thinnings, therefore, in which a range of tree sizes might be removed. Consequently, diameter distribution of the stand before and after thinning and of the trees removed are important characteristics with which managers are concerned. DFCNIGM3 is capable of predicting diameter distribution of the stand and providing detailed information about the future stand structure in a direct manner, in contrast to the stand level disaggregation generally used in New Zealand.

7.1 NEW FEATURES OF THE STUDY

7.1.1 Use of the Type III Extreme Value Distribution.

A novel feature of the research is the use of the type III Extreme Value Distribution, itself a reverse Weibull, for generating the appropriate Reverse Weibull distribution as the means of characterising the stand table. The cumulative distribution of type III Extreme Value Distribution is

$$f(x)=\exp[-(\frac{a_x-x}{b_x})^{c_x}] \qquad -\infty \le x \le a_x \ldots\ldots (7.1.1.1a)$$

$$=1 \qquad x \ge a_x \ldots\ldots\ldots\ldots (5.1.1.1b)$$

with probability density function

$$f(x)=\frac{c_x}{b_x}[(\frac{a_x-x}{b_x})^{c_x-1}]\exp[-(\frac{a_x-x}{b_x})^{c}] \ldots\ldots\ldots\ldots (7.1.1.2)$$

Kuru et al., (1990) used the type I extreme value distribution in their study. It has been shown here, however, that, when the initial distribution assumes the Reverse Weibull, the extreme value distribution to be used is type III rather than type I. This was proved mathematically using the Stability Postulate and Gnedenko conditions. That theoretical basis was then used to build a diameter distribution model, DFCNIGM3, for the Douglas fir plantations in the Central North Island of New Zealand and produced a better fit than the empirical approximation using type I.

7.1.2 Parameter Projection of the Type III Extreme Value Distribution.

Kuru (1989) suggested the use of type I extreme value distribution but he did not mention nor present the equations necessary for its parameter projection. In this study, the parameters of the type III Extreme Value Distribution were projected just as was done for the initial distribution so that any projection made for a future stand condition is also based on the Reverse Weibull Distribution and type III Extreme Value Distribution.

7.1.3 Use of an Extreme Percentile

The reverse Weibull "a" parameter is calculated from the following expression derived from the type III Extreme Value Distribution (7.1.1.1a):

$$a=x=a_x-b_x[-\ln(p_x)]^{\frac{1}{c_x}} \quad \text{.........................} (7.1.3.1)$$

where a_x, b_x and c_x are the location, scale and shape parameters of the type III Extreme Value Distribution and p_x is a percentile of the extreme value distribution derived from

$$p_x = F(x) = 1 - \frac{1}{T(x)} \quad \text{(7.1.3.2)}$$

Theoretically, p_x should be the 100th percentile on the extreme value distribution (Johnson and Kotz, 1970). In practice, though, the 100th percentile might not necessarily produce a good fit. Lower percentiles can be chosen to produce a good fit, but below a certain value one runs the risk that the maximum diameter calculated could be exceeded for the specified age. The extreme percentile proposed in this study sets such a lower limit, within which, one can choose a value of p between the extreme percentile p_x and the 100th percentile. This allows a good fit to be produced yet still ensures that the maximum diameter chosen will not be exceeded within a specified age with p_x probability.

7.1.4 Use of the Hossfeld Function

The Hossfeld function was frequently used as a growth and yield projection equation in this study. There is a non-existent yield at age zero when using the Schumacher equation, which can be regarded as not biologically realistic. At age zero, yield equal to zero is a more logical property and the Hossfeld function has the ability to reflect this condition. When fitting an equation to data sets, the Hossfeld equation will consequently be forced to lie within boundaries set by the upper and lower asymptotes that are entirely appropriate for a plantation population.

7.1.5 Diameter Distribution Model for Douglas Fir Plantations in the Central North Island of New Zealand

Before DFCNIGM 2 and 3 were completed, there was a stand level growth and yield model for Central North Island Douglas fir plantations of reportedly poor reliability, but no diameter distribution model so far for those forests. With DFCNIGM3, future stand structure (e.g. mean diameter, height, and volume in each diameter class at a specified age) of the plantations can be forecasted and management decisions can be made on a much sounder basis. This model, therefore, should be useful for production planning and other management purposes.

7.1.6 Quantification of Disease Impact.

Beekhuis (1978) and Manley (1985) studied the impact of Phaeocryptopus gaeumannii on Douglas fir plantations in New Zealand using graphical techniques. In this study, separate models were developed for the growth and development of remeasured sample plots in diseased and undiseased conditions. This allows growth and yield of diseased and undiseased stands to be simulated and the impact of Phaeocryptopus on growth and yield of Douglas fir to be estimated in a rigorous quantitative way for different stand conditions.

7.2 SAMPLE OUTPUTS

A sample run from DFCNIGM2 is presented below to show the appearance and outputs generated by the model. Initial inputs for the simulation are: site index (30 m), mean top height (16.1 m), age (20), stems/ha (1500), basal area/ha (45 m^2/ha), basal area/ha before and after thinning (50, 45 m^2/ha) and stems/ha before and after thinning (2000,1500), as shown on the top of the table. Projected stand statistics for future ages, namely mean top height, stocking, basal area/ha, mean dbhob, volume/ha and merchantable volume/ha are presented in columns 2 to 7 of the table. thinning specifications and residual stand statistics are given in a separate row immediately after a thinning.

Projected statistics for diseased and thinned stand,   date:11-06-1990
Input summary (S,H,T,N,G,Gb,Nb,Ga,Na): 30.0, 16.1, 20.0, 1,500 45.0, 50.0, 2,000 45.0, 1500

AGE (yrs)	MEAN TOP HEIGHT (m)	STOCKING (N/ha)	BASAL AREA (sq m/ha)	MEAN DBH (QUAD) (cm)	VOLUME (cu m/ha)	MERCHANTA VOLUM (cu m/h
20	16.1	1500	45.0	19.5	275	154
21	16.9	1492	47.7	20.2	305	180
22	17.7	1484	50.2	20.8	335	208
23	18.5	1476	52.7	21.3	366	235
24	19.3	1468	55.0	21.8	397	263
25	20.1	1460	57.3	22.3	428	292
26	20.9	1453	59.4	22.8	459	321
27	21.6	1445	61.5	23.3	490	350
28	22.4	1438	63.4	23.7	521	379
29	23.1	1431	65.3	24.1	553	409
30	23.8	1424	67.1	24.5	584	438

items	values
residual BA/ha:	29.7 sq m/ha
residual N/ha:	400 N/ha
removed in thinning:	
stocking:	1024 N/ha
basal area:	37.4 sq m/ha
mean dbh:	21.6 cm
volume:	323 cu m/ha
merchantable volume:	212 cu m/ha

30	23.8	400	29.7	30.7	261	226
31	24.5	400	31.1	31.5	280	245
32	25.2	399	32.6	32.2	301	265
33	25.8	399	34.0	32.9	321	285
34	26.5	398	35.3	33.6	341	306
35	27.1	398	36.7	34.3	362	326
36	27.7	398	38.0	34.9	383	346
37	28.3	397	39.3	35.5	403	367
38	28.9	397	40.6	36.1	424	388
39	29.4	397	41.8	36.6	445	408
40	30.0	396	43.0	37.2	466	429

7.3 THINNING EFFECTS ON YIELDS

In order to analyze the effect of thinning on yield, three examples of thinning regimes are specified below.

initial condition: site index 30 m; initial age 20 years; initial basal area 45 m^2/ha; initial stocking 1500 stems/ha.

1. Effect of numbers of thinnings on yields:

regime (1):

thin to 35 m^2/ha at age 20;

project to age 30 and thin to 45 m^2/ha;

project to age 40 and thin to 50 m^2/ha;

project to age 70.

regime (2):

same as regime (1) without third thinning.

Outputs of runs based on the specified conditions are presented in the table (7.3.1); where, V(2) and V(3) represent volume production of stand thinned 2 and 3 times and $V_t(2)$ and $V_t(3)$ represent total production of stands thinned 2 and 3 times respectively.

The table shows that final total volume production decreases as the number of thinnings increases.

Table (7.3.1) Effect of number of thinnings on yields

Age	V (2)	V (3)	V_t (2) (m^3 / ha)	V_t (3)	$V_t(2)-V_t(3)$
20	275	275	275	275	0
30	393	393	551	551	0
40	644	540	802	802	0
50	877	757	1035	1019	16
60	1080	950	1238	1212	26
70	1252	1118	1410	1380	30

2. Effect of weight of thinning on yields:

Based on the same initial stand conditions specified in 1. above, another simulation was done with a lighter thinning:

thin to 30 m^2/ha at age 20;

project to age 30 and thin to 40 m^2/ha;

project to age 40 and thin to 45 m^2/ha and project to age 70.

Results of this simulation were then compared with the first regime in 1 above (Table 7.3.2, where, V(1), $V_t(1)$ represent volume and volume production of lightly thinned stands and V(2), $V_t(2)$ represent that of heavily thinned stands). The result showed that both volume and total volume production decreases as weight of thinning increases.

Table (7.3.2) Effect of weight of thinning on yields

Age	V (1)	V(2)	V_t (1) (m^3 / ha)	V_t (2)	$V_t(1)-V_t(2)$
20	275	275	275	275	0
30	393	350	551	532	19
40	540	487	802	770	32
50	757	693	1019	976	43
60	950	881	1212	1164	48
70	1118	1045	1380	1328	52

3. Effect of thinning time on yield:

In this case, a simulation was done by delaying the first thinning for 10 years. The simulated results were compared with the initial regime (no deferral of thinning) based on the same initial stand conditions specified in Table (7.3.1). In Table (7.3.3), V(1) and $V_t(1)$ represent volume and volume production of a stand thinned three times; V(2), $V_t(2)$ represent volume and volume production of a stand treated with the same thinning regime but the thinning was delayed for 10 years.

Table (7.3.3) shows that if thinning is delayed for 10 years, volume production is slightly less. The stand with thinning delayed has a higher total volume production before age 40, but the stand without thinning delay reaches and surpasses this level soon afterwards.

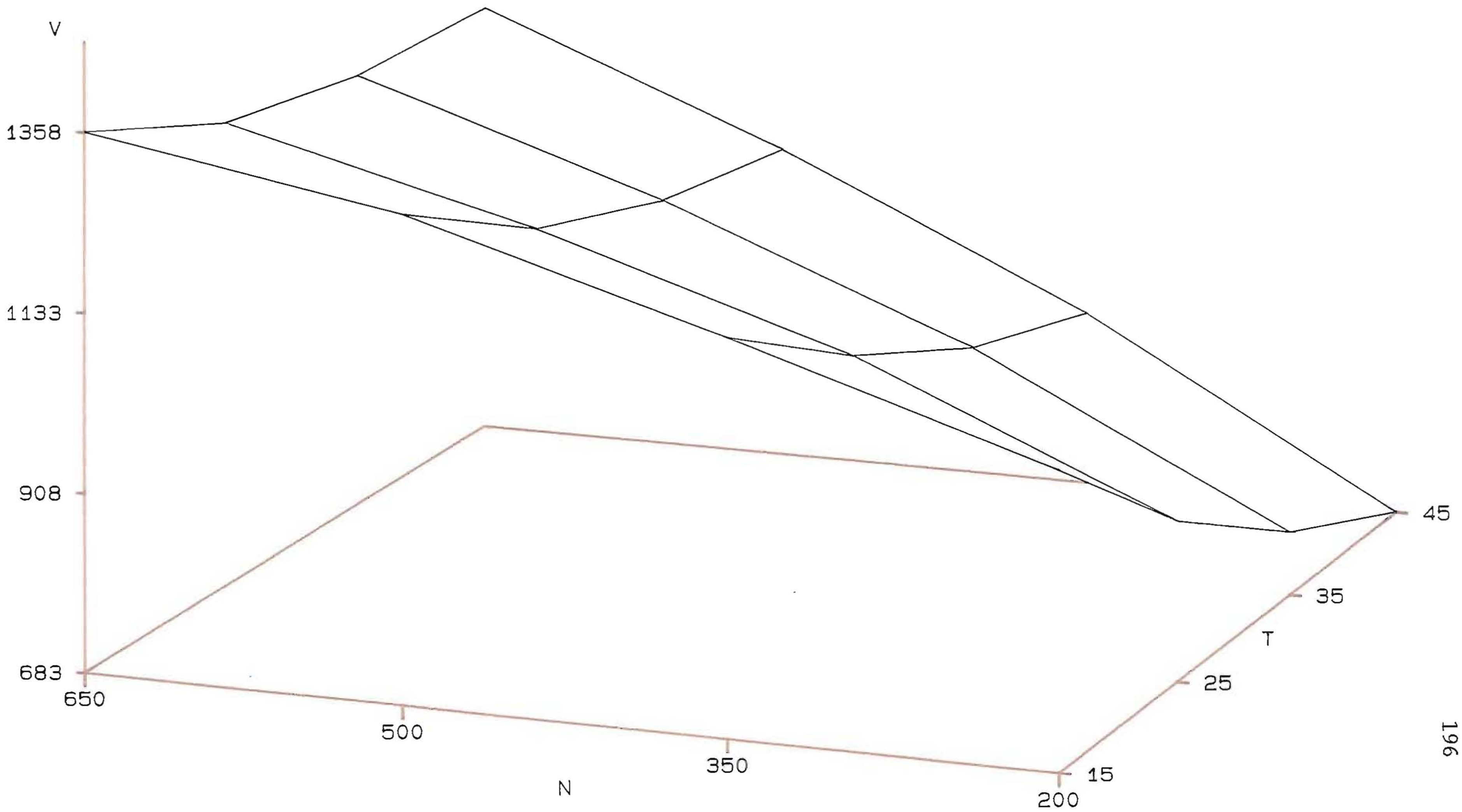
It should be noted that with different initial stand conditions and thinning regimes, results of such analysis

Table (7.3.3) Effect of time of thinning on yields

Age	V (1)	V(2)	V_t (1)	V_t (2)	$V_t(1)-V_t(2)$
			(m^3 / ha)		
20	275	275	275	275	0
30	393	307	551	584	-33
40	540	487	802	807	-5
50	757	620	1019	1013	6
60	950	800	1212	1193	19
70	1118	959	1380	1352	28

would be different as shown in Fig. (7.1), a graph of response surface for a range of regimes. The graph is a result of 16 runs of DFCNIGM2 with initial inputs: site index = 30 (m), age = 15 (years), mean top height = 11.7 (m), basal area = 45 (m^2/ha) and stems/ha = 1500; thinning ages: 15, 25, 35, 45 and residual stems/ha: 200, 350, 500, 650. each simulation was started at age 15, thinned at the age given above and then projected to age 60. For example, with the given initial inputs and thinning age of 25, separate simulation was done for residual stockings of 200, 350, 500 and 650 respectively. Fig. (7.1) was then produced based on those simulation results using SAS.

FIG (7.1) TIME, WEIGHT OF THINNING EFFECT ON YIELDS



7.4 PRODUCTION AND GROWTH LOSS DUE TO THE DISEASE

Yields forecasted using the diseased and undiseased components of DFCNIGM2 can give some indication of the loss of productivity resulting from the presence of Phaeocryptopus gaeumannii. The comparison carried out depends not only on disease but also age, kind and time of thinning. To provide a general indication of the magnitude of the yield difference between diseased and undiseased stands, an average stand condition is outlined below and a comparison made.

Initial stand condition:

initial age 20 years;

site index 30 m;

initial stocking 1500 stems/ha;

initial basal area 45 m^2/ha.

age and thinning conditions:

at age 20 thinned to 35 m^2/ha;

project to age 30 and thinned to 45 m^2/ha;

project to age 40 and thinned to 50 m^2/ha;

project to age 70

Based on the same condition specified above, projections are made for diseased and undiseased stands using DFCNIGM2. Table 7.4.1 shows the differences in projected yields at specified ages. In the table, V_d and V_u represent volume production of diseased and undiseased stand; V_{td} and V_{tu} represent total volume production of diseased and undiseased stands respectively.

Table (7.4.1) Yield differences between diseased and undiseased stands

Age	V_u	V_d	V_u-V_d	V_{tu}	V_{td}	V_{tu}-V_{td}	$100(V_{tu}$-$V_{td})/V_{tu}$
			(m^3/ha)				(%)
20	268	275	-7	268	275	-7	-2
30	511	393	118	664	551	113	17
40	833	540	293	1089	802	287	26
50	1226	757	469	1482	1019	463	31
60	1560	950	610	1816	1212	604	33
70	1830	1118	712	2086	1380	706	34

From the table, in terms of net volume yield, one can roughly say that for a young stand (30 years of age), a diseased stand produces about 23 % less volume than that of the same undiseased stand and it is about 39% less for old stands (60 years of age); in terms of total volume production, the reduction is 17% and 33% respectively. But, in the above simulation, the same amount of basal area / ha has been removed in thinning of diseased as for the undiseased stands. If the same amount of basal area/ha is retained after thinning in diseased and undiseased stands, the difference, in terms of net volume yield, will be smaller. In that case, the difference at age 30 and 60 would be about 15% and 35% respectively, which are the same as concluded by Beekhuis (1978) and Manley (1985).

To model and analyze the disease impact more precisely, we need to know the exact time the disease appeared and its affect on growth over time. Provenance and seedlot records (maps), FRI PSP sample records and studies of disease development had been investigated during this study but, unfortunately, no comprehensive information and useful prediction for the disease was found.

There is a need, therefore, refine the model developed in this study by collecting comprehensive disease information of this species.

7.5 DIFFERENCES OF YIELDS PREDICTED BY DFCNIGM2 AND DFIR

Predictions made using DFCNIGM2 and the model developed by Mountfort (1978) are compared in terms of predicted values and corresponding form factors (Table 7.5.1), where G (2), V(2), F(2) are basal area / ha, volume / ha and form factor predicted from DFCNIGM2; G (m), V (m) and F (m) are basal area / ha, volume / ha and form factor predicted from DFIR; h_{100} is mean top height used to run the two models. The table shows that

(1) predicted stems/ha from DFIR is very high;

(2) predicted basal area/ha from DFIR is higher at an early age and lower at a later age when compared with DFCNIGM2;

(3) predicted volume/ha from DFIR is much lower than that from DFCNIGM2. That is an under-prediction confirmed by the size of the form factor. Calculated form factors

from PSP plot indicates that form factor of Douglas fir plantations in the Central North Island are between 0.3 and 0.5 and seldom go below 0.3. The predicted form factors from Mountfort's model are all below 0.3 throughout the whole range of ages. Intuitively, then, one could say that DFIR under-predicts total stem volume/ha by 500 m^3/ha by age 60 (also merchantable volume/ha by a similar amount). Merchantable volume in DFIR is estimated as some per cent (input by user) of total volume, thus total volume prediction error is necessarily carried through to the merchantable volume.

Table (7.5.1) Comparisons of predictions from DFCNIGM2 and DFIR

AGE	G (2)	G(m)	V(2)	V(m)	h	F(2)	F(m)
20	45.0	45.0	275	195	16.1	0.38	0.27
30	53.7	70.9	467	436	23.8	0.37	0.26
40	72.0	87.3	775	663	30.0	0.36	0.25
50	93.7	98.9	1154	863	34.8	0.35	0.25
60	111.9	108.6	1511	1037	38.3	0.35	0.25
70	126.4	117.4	1816	1195	41.0	0.35	0.25

It is also noted that before about age 40 years, Mountfort's model predicts much higher basal area/ha, but the corresponding volume is much lower. This is further indication of incompatibility, inconsistency and illogicality in that model, as explained in Chapter 1.

7.6 POSSIBLE REFINEMENTS TO DFCNIGM2 AND DFCNIGM3

In order to improve projection precision of DFCNIGM2 and DFCNIGM3, the following refinements are envisaged.

(i) It is possible to work out the time when disease infection starts to affect growth and the corresponding height, diameter and disease affect on growth over time by collecting samples of stem analysis in old stands established before the disease came in. The fit of basal area projection equations can be improved by using a disease index derived from such information.

(ii) further improvement in the goodness-of-fit in the mortality equation needs to be encouraged, though it may not necessarily be easy to achieve;

(iii) goodness-of-fit of some of the equations for the undiseased stands is not as good as that for the diseased stands because of the smaller number of observations available for fitting the equations. If exact time of disease infection can be determined, a precise division between diseased and undiseased stands could have been made. The numbers of observations for the undiseased stands is expected to increase after this division. Consequently, the fits of the equations for both diseased and undiseased stands could be expected to improve. Disease has a major impact on yield and we need better sensitivity on disease history to refine production forecasts.

CHAPTER 8 SUMMARY OF CONCLUSIONS

This study resulted in the development of both a stand and a diameter distribution growth model for Douglas fir even-aged stands in the Central North Island of New Zealand. Both models contain several new features which appear to have led to improved precision and reliability of estimated future production.

The stand growth and yield model, comprising components for projecting growth in mean top height, basal area/ha, total stem volume/ha, merchantable volume/ha, and stems/ha can be described in the following ways.

Separate basal area, volume and merchantable volume equations were obtained for infected and uninfected stands except those for mean top height and mortality which do not distinguish between healthy and diseased stands. The best-fitting equations formed are:

Mean top height

$$h_{100,2}=h_{100,1}\left(\frac{1-\exp(\beta T_2)}{1-\exp(\beta T_1)}\right)^{\beta} \qquad (8.1)$$

Basal area/ ha, diseased stands

$$G_2=\frac{1}{\left(\frac{1}{G_1}\right)\left(\frac{T_1}{T_2}\right)^{\left(\gamma+\frac{\delta}{T_2}\right)}+\alpha\left(1-\left(\frac{T_1}{T_2}\right)^{\left(\gamma+\frac{\delta}{T_2}\right)}\right)+\beta X\left(1-\left(\frac{T_1}{T_2}\right)^{\left(\gamma+\frac{\delta}{T_2}\right)}\right)} \qquad (8.2)$$

Basal area/ha, undiseased stands

$$G_2=\frac{1}{(\frac{1}{G_1})(\frac{T_1}{T_2})^{\gamma}+\alpha(1-(\frac{T_1}{T_2})^{\gamma})+\beta X(1-(\frac{T_1}{T_2})^{\gamma})}\ldots\ldots\ldots\ldots(8.3)$$

Basal area/ha after a thinning specified in stems/ha removed

$$G_a=\alpha G_b^{\beta}(1-(1-\frac{N_a}{N_b})^{\gamma})^{\delta}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.4)$$

Total stem volume/ha, infected stands

$$V_t=\alpha+\beta h_{100}+\gamma G+\delta h_{100}G\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.5)$$

Total stem volume/ha, healthy stands

$$V_t=\beta h_{100}+\gamma G+\delta h_{100}G\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.6)$$

Merchantable volume/ha

$$V_m=\alpha V_t^{\beta}\exp(\gamma\frac{d_{15}}{d^{\delta}})\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.7)$$

Mortality

$$N_2=N_1(\frac{T_2}{T_1})^{\beta(T_2-T_1)(\alpha G+\gamma S+\delta d_g)X}\exp((T_2-T_1)(\alpha G+\gamma S+\delta d_g)X)\ldots\ldots(8.8)$$

Site index, mean top height at age 40 years for Douglas fir, can be derived from setting T_2 = 40 years in the first equation. About 97 % of the observations used to fit the model could be predicted within ± 1 m of the actual values. Mean predicted error is 0.42 m. Unlike the existing site index equation for Central North Island Douglas fir developed by Burkhart and Tennent (1977), this adaptation in 8.1 allows site index to be directly derived.

The rarely used Hossfeld function was found to be the best means of projecting basal area/ha for this data set. This equation has one more desirable property than the Schumacher equation possesses, namely, when stand age $T = 0$, yield $Y = 0$, whereas in contrast, when $T = 0$, Y is not defined in the Schumacher equation. When fitted to data, this yield curve is therefore forced to fall within the lower and upper bounds of a data set, zero and maximum yield of the stand. All the residuals for the Hossfeld equation lie within ± 2.5 m^2/ha for the infected stands and 2.9 m^2/ha for healthy stands. Mean predicted error is 0.53 m^2/ha for infected stands and 0.83 m^2/ha for healthy stands.

Equation (8.3) can be used to estimate basal area/ha after thinning, so that thinning production and residual crop yields can be predicted in steps to coincide with changes in stocking. To make use of this capability of the model, input is required of basal area/ha and/or stems/ha before thinning. For estimating basal area after thinning with a specified stems/ha removal, 95 per cent of the residuals lie within ± 5 m^2/ha. Specification of basal area/ha removal is therefore preferred for forecasting production after thinning.

The total stem volume equation has nearly all its residuals within ± 43 m^3/ha for diseased stands and 34 m^3/ha for undiseased stands. Mean predicted error will be within 6.7 m^3/ha for diseased stands and 7.1 m^3/ha for healthy stands. The merchantable volume equation has most of its residuals within ± 4 m^3/ha for diseased and healthy stands and predicted errors are 0.1 m^3/ha and 0.6 m^3/ha respectively. The predicted errors from total volume should be included in the predicted errors of merchantable volume because the estimation of merchantable volume is based on total volume.

The mortality equation in DFCNIGM2 reduced the residual sum of squares by 37 per cent compared with the equation used in DFCNIGM1. In forecasting future stems/ha, 95 per cent of the residual lie within 70 stems/ha.

Thus, the stand model produces accurate and precise stand level yield forecasts for each of the types of Douglas fir stands in the Central North Island of New Zealand. This also ensures accurate and precise stand-level forecasts from the diameter distribution model, which disaggregates the stand variable into dbhob classes.

Diameter distributions in this study were generated through use of the reverse Weibull distribution. This allows the maximum diameter to be used as the location, a stand statistic which is much easier to fit, because of its closer correspondence to changes in stocking than minimum diameter.

The c parameter of the reverse Weibull distribution can be estimated through (8.9) using an iterative procedure; b can be obtained by solving the following:

In terms of equations (8.9) and (8.10), the standard

$$\frac{s}{\bar{x}}=\frac{\left(\Gamma(1+\frac{2}{c})-\Gamma^2(1+\frac{1}{c})\right)^{\frac{1}{2}}}{\Gamma(1+\frac{1}{c})}=\hat{c}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.9)$$

$$\bar{x}=b^{\frac{1}{c}}\Gamma(1+\frac{1}{c})\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.10)$$

deviation was projected using the Hossfeld equation but modified somewhat into:

$$d_{s2}=\frac{1}{\frac{1}{d_{s1}}(\frac{T_1}{T_2})^{\delta}+\alpha d_{max1}(1-(\frac{T_1}{T_2})^{\delta})+\beta d_{min1}(1-(\frac{T_1}{T_2})^{\delta}+\gamma k(1-(\frac{T_1}{T_2})^{\delta}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.11)$$

For this equation, 95% of the residuals for diseased stands lie within 0.70 cm with a predicted mean absolute error of 0.10 cm while 95% of the residuals for healthy stands lie within 0.63 cm with a predicted mean absolute error of 0.17 cm.

Quadratic mean diameters are obtained from:

$$d_{\bar{g}}=200\sqrt{\frac{G}{\pi N}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.12)$$

and arithmetic mean diameter from the relationship

$$s^2=\frac{\sum d_g^2-(\sum d)^2/N}{N}, \qquad d^2=d_g^2-s^2\ldots\ldots\ldots\ldots\ldots\ldots(8.13)$$

With the standard deviation of diameters and the mean diameter estimated, c and b parameters of the reverse Weibull distribution are readily solved.

Previous work has used the type 1 extreme value distribution to locate the Weibull parameter a. In this study, it is shown that type III rather than type I extreme value distribution should be employed.

The existence of such an extreme value distribution can be tested by the Gnedenko conditions:

(1) *for* *I*: $\lim_{n=\infty} n[1-F(u_n+y/\alpha_n)] = e^{-y}$ ....(5.14)

(2) *for* *II*: $\lim_{x=0} \frac{1-F(x)}{1-F(cx)} = c^k$ ....(5.15)

(3) *for* *III*: $\lim_{x=-0} \frac{1-F(cx+\omega)}{1-F(x+\omega)} = c^k$ ....(5.16)

The procedure for testing the type III

$$\lim_{x\to-0} \frac{1-F(\alpha x)}{1-F(x)} = \lim_{x\to-0} \frac{1-\exp[-(\frac{-\alpha x}{b})^c]}{1-\exp[-(\frac{-x}{b})^c]} \qquad let \quad z = x/b$$

$$= \lim_{Z\to-0} \frac{1-\exp[-(-\alpha Z)^c]}{1-\exp[-(-Z)^c]}$$

$$= \lim_{Z\to-0} \frac{-e^{-(-\alpha Z)^c}.-c(-\alpha Z)^{c-1}.-\alpha}{-e^{-(-Z)^c}.-c(-Z)^{c-1}.-1}$$

$$= \lim_{Z\to-0} \frac{e^{-(-\alpha Z)^c}.\alpha^{c-1}.\alpha}{e^{-(-Z)^c}}$$

$$= \alpha^c$$

............................(8.17)

According to the symmetry principle, the Weibull distribution is the smallest extreme value distribution when one works with the smallest diameters. In other words, the procedure proposed here can also be used in the case where conventional Weibull distribution function is used.

When working with the three parameter reverse Weibull distribution, the equation for calculate the "a" parameter is

$$x = a_x - b_x[-\ln(p)]^{1/cx} \quad \text{.........................(8.18)}$$

Where $p = p_x = F(x)$, called extreme percentile, is a percentile on the type III Extreme Value Distribution:

$$p=p_x=F(x)=1-\frac{1}{T(x)} \quad \text{..............................(8.19)}$$

calculated on the basis of return period:

$$p=T(x)=\frac{1}{1-F(x)} \quad \text{...............................(8.20)}$$

a_x, b_x and c_x are the location, scale and shape parameters of the type III Extreme Value Distribution and T(x) is the return period. This procedure allows the maximum diameter of the stand to be estimated free of bias.

The parameters of the extreme value distribution have been projected for future stand conditions. This means that at any time the reverse Weibull "a" parameter will be estimated through using the type III Extreme Value Distribution in combination with the extreme percentile and it will not be exceeded for a specified age. The parameter projection equation form for the extreme value distribution is

$$d_{xmax2}=\frac{1}{(\frac{1}{d_{xmax1}})(\frac{T_1}{T_2})^{\gamma}+\alpha(1-(\frac{T_1}{T_2})^{\gamma}} \quad \text{................(8.21)}$$

Prediction of maximum, mean and standard deviation of extreme largest diameter were most precise, despite the small number of observations (30 to 170).

Height corresponding to the mid-point diameter was projected by the following equation

$$h=\alpha+\beta h_{100}+\gamma dbh \quad (8.22)$$

which gives a good fit in terms of residual patterns but has high extreme values of the residuals. Predicted error is 1.22 m for the diseased stands and 1.14 m for healthy stands. One merit of this equation is that it reflects the site variation among different forests through inclusion of the variable h_{100}.

In general, all the components of the diameter distribution model provide good fits and will produce precise prediction of future stand statistics.

The parameter recovery procedure was used to estimate the Weibull parameters, b and c, so that the forecasts are compatible between the whole stand and the diameter distribution models and there is consistency, therefore, among various yield estimates.

CHAPTER 9 RECOMMENDATIONS

ACKNOWLEDGEMENTS

REFERENCES

APPENDICES

9.1 RECOMMENDATIONS

9.1.1 Conditions under Which the Models Apply

The models apply to the Kaingaroa region which included Kaingaroa, Pureora, Waimihia and Whirinaki forests and to the Douglas fir plantations aged between 15 and 70 years. Users who might like to use the model to make production forecasts for Douglas fir plantations outside these limits should conduct some independent verification before accepting the results.

9.1.2 Inputs to the Models

The initial inputs of mean top height, a measurement, is preferred to site index, an estimate, which could adds its estimated error to yield prediction. For a thinning specification, input of basal area/ha is preferred to the stems/ha because the former is the variable used to calculate volume production. All inputs should be accurate to one decimal place except initial stems/ha which is integer.

For DFCNIGM1 and DFCNIGM2, inputs required are similar to most existing growth and yield models in New Zealand. For DFCNIGM3, however, additional inputs are required: maximum, mean and standard deviation of the largest diameters. These inputs should be calculated in this way: the sample to be used to supply the inputs are sorted by age and the maximum, mean, standard deviation are calculated for each age. Inputs of the maximum values should be those referring to the starting age. For example, if the model is run with a start age of 25, then the inputs of maximum, mean and standard deviation should be those calculated for age 25.

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8.4 APPENDICES

Appendix 1 reprint of Woollons, R.C., Whyte, A.G.D. and Liu Xu (1990). The Hossfeld function: an alternative model for depicting stand growth and yield. Journal of the Japanese Forestry Society.

THE HOSSFELD FUNCTION : AN ALTERNATIVE MODEL FOR DEPICTING STAND GROWTH AND YIELD AND GROWTH

R C Woollons[1] , A G D Whyte[2] and Liu Xu[3]

ABSTRACT

This paper (1) sets out two forms of a sigmoid growth equation called the Hossfeld function, (2) examines its basic properties and (3) assesses its utility for modelling growth and yield of even-aged forest stands. A small data set comprising several re-measurements of mean top height and net basal area/ha in sixty Douglas fir permanent sample plots in the Central North Island of New Zealand was used to test the fit of the Hossfeld function to real data. The fit of mean top height on age was slightly better for the polymorphic than for the anamorphic form. When compared with the polymorphic log-reciprocal and anamorphic Chapman-Richards, the polymorphic Hossfeld function for mean top height produced a slightly more precise fit and equally satisfactory residual patterns. Similarly, net basal area/ha was equally well modelled over time by the Hossfeld, log-reciprocal and modified Gompertz functions. The Hossfeld function, therefore, should be recognised by researchers as an equation well worth considering in the development of compatible growth and yield models.

INTRODUCTION

Over the past twenty-five years, forest scientists have adopted a variety of sigmoid-shaped functions to model growth and yield of forest stands through time. Clutter (1963), for example, utilised the log-reciprocal equation suggested by Schumacher (1939), to model basal-area development of unthinned *Pinus taeda* L. This model proved to be most useful for forest growth modelling, and has been used extensively with many species [see, for example, Bennett (1970b), Leak *et al*. (1970), Perala (1971), and Woollons and Hayward (1985)]. Occasionally, however, some scientists have chosen alternative yield functions. Pienaar and Turnbull (1973) utilised the Chapman-Richards equation to predict basal area of *Pinus elliottii* Engelm. from South African CCT

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permanent sample plots [O'Connor (1935)], while Nokoe (1978) used a modification of the Gompertz yield function to model volume per hectare in three species (<u>Thuja plicata</u>, <u>Pinus contorta</u>, and <u>Pseudotsuga menziesii</u>). Then again, Yang <u>et al</u>. (1978) advocated the use of the Weibull function to produce flexible growth curves, while Smith and Kozak (1984) extolled the utility of the Schnute (1981) model. Ferguson (1983) reviewed the performance of several yield models for radiata pine in Australia.

Researchers in Japan have been much concerned with theoretical aspects of growth and yield modelling of unthinned or self-thinned stands. For example, Minowa (1982, 1983), Naito (1984) and Sweda (1984) have reviewed various optional forms of characterising whole stand approaches while Tanaka (1987) has concentrated on a diameter class technique. This emphasis on theory appears to be largely a consequence of insufficient field data to test the theory. Theoretical and practical considerations need to interact, however, to gain a useful perspective. This paper illustrates, therefore, the importance of such interactions on desirable forms of growth models.

All these sigmoid functions mentioned above are linked in that each can be derived from linear first-order differential equations. Umemura (1984), however argued, again theoretically, the case for growth models based on second order differential equations, and developed two yield functions, as a result of considering distinct or coincident roots. This concept is certainly worth exploring further, but attention here is focussed for the present on the use of only first-order differential equations. A later contribution will be directed at Umemura's hypothesis.

In this contribution, we study an equation not known extensively in Western countries, namely the Hossfeld function [Peschel (1938) and Prodan (1968)]. Some of its statistical properties are developed, and its utility is demonstrated by fitting it to a forest dataset. The results are discussed, and the utility of the function in relation to some commonly employed others is reviewed.

HOSSFELD EQUATION : STATISTICAL PROPERTIES

The Hossfeld function is given by

$$Y = (\alpha T^{\gamma}) / (\alpha\beta + T^{\gamma}) \qquad (1)$$

where in (1)

$Y =$ yield at time T
$\alpha, \beta, \gamma =$ parameters (to be estimated)

Evidently, when $T = 0$, then $Y = 0$, and when $T \to \infty$, then

$$Y = \lim_{T \to \infty} (\alpha T^{\gamma}) / (\alpha\beta + T^{\gamma})$$

$$= \lim_{T \to \infty} \alpha\gamma T^{(\gamma-1)} / (\gamma T^{(\gamma-1)})$$

$$= \alpha \qquad \text{[by L'Hopitals rule]}$$

Differentiating (1) with respect to T gives the <u>growth</u> equation

$$dY / dT = \alpha\beta\gamma Y / T(\alpha\beta + T^{\gamma}) \qquad (2)$$

Differentiating (2) again, to give the second differential, d^2Y / dT^2, and setting the resultant expression to zero, produces a point of inflexion:

$$Y_{inflex} = \alpha(\gamma-1) / 2\gamma \qquad \text{at } T = ((\alpha\beta(\alpha-1)) / (\alpha + 1))^{\frac{1}{\gamma}} \qquad (3)$$

Thus, equation (1) is shown to be a sigmoid growth curve, which goes through the origin, with an upper asymptote, α. From (2), we note that growth is postulated to be a function of current yield, Y, inverse of age, T, and a maturation factor, $(\alpha\beta + T\gamma)^{-1}$. The yield equation, therefore, makes good biological as well as mathematical sense.

Equations such as (1) can be enhanced by expressing them as difference equations [Clutter (1963); Clutter <u>et al</u>. (1983), Borders <u>et al</u>.

(1984)]. For a nominated yield, Y_1, at time T_1. Equation (1) can then be re-expressed as:

$$Y_2 = f\,(Y_1, T_1, T_2 \mid \alpha,\beta,\gamma) \qquad (4)$$

Two forms of (4) are available, anamorphic or polymorphic [Borders et al. (1984)]. With the latter, the shape parameter β is postulated to be site specific, and eliminated from equation (4) in the following manner:

$$Y_1 = \alpha T_1\gamma / (\alpha\beta_i + T_1^{\gamma}) \text{ and } Y_2 = \alpha T_2 / (\alpha\beta_i + T_2^{\gamma}) \qquad (5)$$

Eliminating β_i from (5) leads to the difference equation

$$(1/Y_2) = (T_1/T_2)^{\gamma}(1/Y_1) + (1/\alpha)[1-(T_1/T_2)^{\gamma}]$$

or

$$Y_2 = 1 / \{(T_1/T_2)^{\gamma}(1/Y_1) + (1/\alpha)\,[1-(T_1/T_2)^{\gamma}]\} \qquad (6)$$

Alternatively an anamorphic form of (4) can be constructed by regarding the asymptote α to be site specific, which gives the difference equation:

$$(1/Y_2) = (1/Y_1) + \theta(1/T_2^{\beta} - 1/T_1^{\beta})$$

or (7)

$$Y_2 = 1/((1/Y_1) + \theta(1/T_2^{\beta} - 1/T_1^{\beta}))$$

where in (7)

θ = a function of β and α

METHODS

Plantation forests in New Zealand are dominated by the conifer Pinus radiata, but there are also significant holdings of, for example, Eucalypts and Douglas-fir [Pseudotsuga menziesii]. A Douglas fir data set was chosen to test the utility of the Hossfeld projection equations for both mean top height and net basal area/ha developed above. Mean top height estimates, defined as the average height of the largest 100 stems/ha by diameter, and net basal area/ha were available from 60 permanent sample plots. A summary of relevant plot statistics is given in Table 1

Table 1: **Summary of plot data**

	Mean	Min.	Max.	
Age (measured)	35	13	61	(years)
Top height	27.7	10.6	43.9	(m)
Site index*	31.7	19.5	38.8	(m)
Plot stocking	952	158	2411	(stems/ha)
Net basal area	59.9	15.9	110.1	(m^2/ha)

* defined as plot mean top height, at age 40

A plot of top-height development against stand age for all data is given in Figure 1

[TAKE IN FIGURE 1]

Equations (6) and (7) were then fitted to the relevant data using the NLIN procedure of the SAS statistical system (SAS Inst [1985])

RESULTS AND DISCUSSION

Parameter estimates for the two models of mean top height were:

<u>Polymorphic form</u> (equation 6)

$\alpha = 0.014\,733$ s.e. $= \pm\, 0.001\,013$

$\gamma = 1.489\,727$ s.e. $= \pm\, 0.074\,146$

Residual mean square, $\sigma^2 = 0.336\,04$

A plot of residual data values against predicted figures [equation 6] is given in Figure 2; the data are evidently well-behaved, with no sign of bias or systematic patterns, indicative of an excellent goodness-of-fit (Draper and Smith [1981], Chapter 3).

[TAKE IN FIGURE 2]

Anamorphic form (equation 7)

$\theta = 1.910\ 298$ s.e. $= \pm\ 0.234\ 505$

$\beta = 1.244\ 341$ s.e. $= \pm\ 0.048\ 215$

Residual mean square, $\sigma^2 = 0.375\ 06$

A plot of residual against predicted values [not given here] gives a generally random appearance, but less satisfactorily so than in Figure 2 above.

It is thus clear that both projection equations, (6) and (7) give excellent models of top height development, but equation (6), the polymorphic form, is somewhat superior. This can be substantiated by comparing the respective residual mean squares, where that associated with equation 6, is 10% lower in value. Nevertheless, either form has provided a satisfactory fit.

The projection form (6) can be transformed moreover, to generate a set of site-index curves; by definition, site index, S, is that mean top height which is achieved at age 40. By substituting $T_2 = 40$, and S (site index) = $\bar{h}_{100,2}$ we have

$$1/S = 1/\bar{h}_{100,2}(T_1/40)^{\gamma} + (1/\alpha)\,(1-(T_1/\ 40)^{\gamma})$$

or

$$(1/\bar{h}_{100,2}) = [(1/S) - (1/\alpha)\,(1-(T_1/40)^{\gamma})]\,(T_1/40)^{-\gamma} \qquad (8)$$

Figure 3 shows a set of site-index curves, resultant from equation (8); convergence to a common asymptote, indicative of a polymorphic function, is clearly depicted.

[TAKE IN FIG.3 HERE]

In the above analysis, no tabulation of analyses of variance, or t-statistics (associated with parameters) is provided because of serial-correlation existing in the data, obtained from repeated measurements of identical sets of trees; as such, error terms will be underestimated [Davis and West (1981), Ferguson and Leech (1981)]. Nevertheless, this complication has no significant effect in practice; careful use of graphical plots as assayed here allows goodness-of-fit assessment, while in any event the t-statistics are usually irrelevant, since the majority of parameters tested are established by *a priori* knowledge [Woollons and Hayward (1985)].

A reasonable fit of net basal area/ha for this data set was also achieved with equation 6. The estimated coefficients, α = 118.6875 ± 2.3650 γ = 2.045 73 ± 0.057 77, and σ^2 = 1.3189 were precise enough: residual patterns, moreover, showed a reasonably satisfactory spread, with every indication that improvements to the fit could be made through the addition of the usual kinds of other explanatory variables, such as stocking and thinning index, [see Weir (1989)].

The Hossfeld function is thus shown here to be an excellent model for depicting top height development and basal area/ha with the data at hand. At no time, however, do we claim it to be an optimal equation for modelling stand growth in general; alternative models may well give equal precision, or the function may not perform as well with alternative species or data.

Two other frequently used equations were fitted to the mean top height data for comparative purposes.

(1) The log reciprocal yield equation (Schumacher, 1939),

$$Y = \exp\left(\alpha + \beta / T^{\gamma}\right) \qquad (9)$$

can be modified to the following projection form,

$$\bar{h}_{100,2} = \bar{h}_{100,1}\left((T_1 / T_2)^{\gamma}(\exp(\alpha(1-(T_1/T_2)^{\gamma}))\right) \qquad (10)$$

(2) The Chapman-Richards has the yield form (Clutter et al., 1984)

$$Y = \alpha\,(1 - \exp(-\beta T))^{(1-\gamma)^{-1}} \qquad (11)$$

and a corresponding anamorphic projection form,

$$\bar{h}_{100,2} = \bar{h}_{100,1}\;((1 - \exp(-\beta T_1))/(1 - \exp(-\beta T_2)))^{(1-\gamma)^{-1}} \qquad (12)$$

The term "exp" represents exponential function.

The fitted coefficients, their standard errors and the model residual mean square are shown in Table 2.

Table 2: Comparison of Hossfeld, Log-reciprocal and Chapman-Richards Equations for estimating mean top height

Function	α	β	γ	σ^2
Hossfeld	0.014 73 ± 0.001 01	-	1.489 72 ± 0.074 15	0.336 04
Log-reciprocal	5.113 ± 0.240 02	-	0.477 59 ± 0.062 09	0.35002
Chapman-Richards	-	0.029 64 ± 0.003 08	1.493 41 ± 0.087 39	0.337 13

Residual patterns for the Schumacher and Chapman-Richards equations are similar to those for the Hossfeld function. The Hossfeld function, however, provides a marginally better fit than either of the other two to this particular set of mean top height data.

Its utility is not confined simply to modelling height growth, however. Zeide (1988 pers. comm.) has, for example, found that it provides a good fit to diameter growth of individual trees. We have compared here its performance in predicting net basal area/ha growth with the log-reciprocal and Gompertz functions. The yield and projection forms for the last one are respectively:

$$Y = \exp(\alpha \exp(-\gamma T - {}^1/_2\, \delta T^2) \tag{11}$$

$$G_2 = \exp(\log G_1 \exp(-\alpha\,(T_2 - T_1) + \delta\,(T_2^2 - T_1^2)) + \alpha\,(1 - \exp(-\gamma\,(T_2 - T_1) + \delta\,(T_2^2 - T_1^2)))) \tag{12}$$

The fits for all three models do not differ to any great extent, as can be seen from Table 3.

Table 3: Comparison of Hossfeld, Log-reciprocal and Gompertz Functions for Estimating Net Basal Area/ha.

Function	α	β	γ	δ	σ^2
Hossfeld	118.6875 ± 2.3649	2.0457 ± 0.0578	-	-	1.318
Log-reciprocal	5.0208 ± 0.0394	0.9989 ± 0.03441	-	-	1.2736
Gompertz	4.9406 ± 0.0438	-	0.08485 ± 0.00389	0.000707 ± 0.000038	1.2873

The log-reciprocal equation performs slightly better than either of the other two, but all three functions are amenable to the addition of further predictor variables and other types of formulation (see Weir, 1989) which would undoubtedly sharpen up their precision. As it is, the standard error of estimate for basal area is less than 2 m^2/ha for all three, a most acceptable level of precision. As for mean top height, use of the Hossfeld function for modelling net basal area/ha should be seriously considered.

We believe it is unrealistic to expect a unique sigmoid function to perform consistently better than others with forest growth and yield data. A more rational approach is to be aware of the existence of several candidate equations, and to explore their utility with data pertaining to individual circumstances. The Hossfeld function provides one viable alternative that should be considered.

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FRI/INDUSTRY RESEARCH COOPERATIVES

STAND GROWTH MODELLING COOPERATIVE

FOREST RESEARCH INSTITUTE
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ROTORUA

DOUGLAS FIR CENTRAL NORTH ISLAND GROWTH MODEL

Liu Xu

REPORT NO. 17 MAY 1990

Executive Summary

DFCNIGM2 is a compatible growth and yield model developed by Mr Liu Xu at the University of Canterbury for simulating growth and yield of even-aged stands of Douglas Fir growing in the Central North Island of New Zealand. The data consist of over 2500 plot measurements from 244 permanent sample plots located in Kaingaroa, Whakarewarewa, Waimihia, Whirinaki, Horohoro, Karioi and Pureora forests. A wide range of ages (10 to 90 years), stockings (200 to 7000/ha), altitudes (200 to more than 900 m) and thinning histories (up to 3) was available for analysis.

Preliminary investigations indicated that growth and yield trends should be characterised individually for three localities (Whakarewarewa, Karioi and Kaingaroa + the rest), two levels of *Phaeocryptopus* infection (diseased or not) and two thinning histories (thinned or not), but that there was no justification for differentiating groups on the basis of altitude nor of initial stocking. As the disease is well established throughout the Central North Island, the branch specifying no disease should be interpreted only as a guide to what growth potential was and could be. It should not be used for today's forecasting.

Each branch through the model has a specific set of equations for:

net basal area/ha on age;

total stem volume/ha in terms of height and basal area;

merchantable volume/ha in terms of total stem volume; except for mortality and mean top height, equations are the same for all branches.

From inputs to an IBM compatible personal computer of starting age, mean top height (or site index), basal area/ha, stems/ha and indices of thinning and disease histories, growth simulations with or without further thinning can be conducted. At any age (but preferably between 15 and 70 years) reliable forecasts of mean top height, stocking, basal area/ha, mean dbhob, total stem volume/ha and merchantable volume/ha can be output on the screen and the printer. Separate estimates are provided for thinning removals and main crop residuals for any specified age of thinning.

The individual equations take the following form

Top Height :

$$h_{100,2} = h_{100,1}((1-\exp(-\alpha T_2))/(1-\exp(-\alpha T_1)))^{\beta}$$

Basal Area:

(a) unthinned stands

$$G_2 = 1/((1/G_1)(T_1/T_2)^{\beta} + \alpha(1-(T_1/T_2)^{\beta}))$$

(b) thinned Stands

$$G_2 = 1/((1/G_1)(T_1/T_2)^{\Gamma} + \alpha(1-(T_1/T_2)^{\Gamma}) + \beta X(1-(T_1/T_2)^{\Gamma}))$$

(c) after a thinning specified in stems/ha removed

$$G = \alpha G_o^{\beta}(1-(1-N/N_o)^{\Gamma})^{\delta}$$

Total Stem Volume:

(a) diseased Stands, thinned

$$V_t = \alpha+\beta G+\Gamma h_{100}+\delta G h_{100}$$

(b) diseased, unthinned stands

$$V_t = \beta G+\Gamma h_{100}+\delta G h_{100}$$

(c) no disease

$$V_t = \alpha+\delta G h_{100}$$

Merchantable Volume:

$$V_m = \alpha V_t^{\beta}\exp(-\Gamma(15/d_g^{\delta}))$$

Mortality:

$$N_2 = N_1(T_1/T_2)^{\beta(T2-T1)(\alpha\Delta G+\Gamma S)}\exp((T2-T1)(\alpha\Delta G+\Gamma S))$$

Where, in the standard IUFRO notation,

$h_{100,i}$: mean top height at age T_i ($i > 0$);
T_i: stand age;
G_i: basal area/ha at age T_i ($_i > 0$);
G_o: basal area/ha before thinning;
ΔG: periodic mean annual increment in basal area/ha;
N_i: stems/ha at age T_i ($_i > 0$);
V_t: total stem volume/ha;
V_m: merchantable volume/ha to 15 cm top;
α, β, Γ, δ are parameters estimated by non-linear least squares or weighted least squares (PROC NLIN or PROC REG in the SAS package).

Site index, mean top height at age 40 years for Douglas Fir, can be derived from setting T_i = 40 years in the first equation. All of the residuals lie within ± 2 m, only one equation for all kinds of stands being needed. This analysis confirms the good estimation of the Burkhart and Tennent equations for Douglas Fir, but the one developed here is to be preferred because, in addition to being a good fit, it allows site index to be derived explicitly and more easily.

The various basal area/ha equations are also very reliable, being unbiased and with all of the residuals lying within ± 2 m^2/ha for thinned stands and ± 3 m^2/ha for unthinned stands. For estimating basal area after thinning with a stems/ha removal, 95 per cent of the predictions lie within ± 5 m^2/ha. Thinnings should therefore be specified preferably in terms of basal area/ha removed.

The total stem volume equation has nearly all its residuals within ± 20 m^3/ha for diseased, thinned stands; ± 40 m^3/ha for diseased, unthinned stands; and ± 30 m^3/ha for undiseased stands. The merchantable volume equation has most of its residuals within ± 23, 44 and 33 m^3/ha for diseased thinned, diseased unthinned and undiseased stand respectively.

Thus, DFCNIGM2, produces accurate and precise yield forecasts for each of the types of Douglas Fir stand in the Central North Island on an IBM PC compatible using the same sorts of inputs as FRI radiata pine growth models.

Appendix 3 Reprint of Liu Xu, Wood, G.R., Whyte, A.G.D. and Woollons, R.C. (1991). Stand table prediction with Reverse Weibull and extreme value density functions: some theoretical considerations. For. Ecol. & Mang. (1991).

Short Communication

STAND TABLE PREDICTION WITH REVERSE WEIBULL AND EXTREME VALUE DENSITY FUNCTIONS: SOME THEORETICAL CONSIDERATIONS

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ABSTRACT

A recent paper by Kuru et al. (1991)[3] used the Reverse Weibull and Extreme Value density functions to generate stand tables for even aged plantations. These authors employed the Type I form of the extreme probability density function to fix a, the location parameter of the Reverse Weibull distribution. They should have used the Type III form and this note explains why. Additional work described here shows that an exact sampling distribution for largest values drawn from a Reverse Weibull is available, with location and shape parameters identical to the parent distribution. The scale parameter is shown to be partially dependent on n, the sample size, for any age. Further work is required to ascertain how n should be best evaluated.

INTRODUCTION

The Weibull density function is very well known to forest growth-modellers as a flexible equation to depict and project diameter distributions [Bailey & Dell (1973)]. Recently, Kuru et al. (1991) suggested reversing the distribution to give

$$f(x) = (c/b)((a-x)/b)^{c-1}\exp[-((a-x)/b)^{c}] \quad \text{for } -\infty < x \leq a \qquad (1)$$

where in (1)

a, b, c are location, scale and shape parameters, respectively.

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[3] Utility of Reverse Weibull and Extreme Value density functions to refine diameter distribution growth estimates. For.Ecol.&Mang (1991).

This enables us to estimate a, the location parameter, as a function of maximum diameter, as opposed to the minimum, since the latter is very difficult to model with any precision [see for example, Clutter & Belcher (1978); Feduccia et al (1979)].

Kuru et al. also suggested adjusting the estimate of a by utilising an extreme value distribution:

$$f(x) = (1/\theta)\exp[-(x-\varepsilon)/\theta]\exp[-\exp(-(x-\varepsilon)/\theta)] \tag{2}$$

and choosing a percentile from (2) which minimises the residual error after fitting (1) to plot data.

Recent work by the first two authors here, has shown that equation (2) above is inappropriate, if regarded as the limiting distribution for maxima in samples from Reverse Weibull density functions. In this note, we describe the derivation of the correct asymptotic distribution, and also show that an exact sampling distribution is available. Implications of substituting these in Kuru's model are discussed.

EXTREME VALUE DISTRIBUTIONS

The study of extreme values is a branch of order statistics. A comprehensive exposition of the topic is given by Gumbel (1958), while shorter accounts have been given by, for example, Kendall & Stuart (1958) and Johnson & Kotz (1970).

Derivation of the cumulative distributional form, G, of the largest value of a sample of size n drawn from a parent distribution, F, relies on the so-called "Stability Postulate". This postulate, which not every F will satisfy, is the commonsense statement that the distribution of the largest value in N samples, each of size n, will tend to the same distributional form with increasing n as that of the largest value in samples of size n. It dates back to Frechét (1927), and for each N leads to the relationship:

$$[G(x)]^N = G(a_N x+b_N) \tag{3}$$

for constants a_N and b_N, dependent on N>1.

A Reverse Weibull parent distribution F is bounded above, so it is evident that in this case, if G exists, the scale factor a_N cannot equal 1 : bunching of the extreme value distribution must occur immediately beneath the upper bound. In this case Gumbel (1958, pp 159-162) shows that G must have, coincidentally, the form of a Reverse Weibull distribution with a "Type III" asymptote.

In this paper the parent distribution itself is the Weibull. Does the Stability Postulate hold in this case? Gnedenko (1943) has given necessary and sufficient conditions for an F, bounded above, to satisfy the postulate. A parent distribution F, with upper bound of 0, will have a Type III asymptotic exreme value distribution if and only if

$$\lim_{x \to 0^-} \quad 1 - F(\alpha x)/(1 - F(x)) = \alpha^k \tag{4}$$

for any $\alpha > 0$ and some $k > 0$. For the Reverse Weibull,

$$F(x) = \exp\{-((a-x)/b)^c\} \tag{5}$$

and without loss of generality setting a = 0 and b = 1, we have, substituting in (4)

$$\lim_{x \to 0^-} \quad (1-\exp\{-(-\alpha x)^c\})/(1-\exp\{-(-x)^c\})$$

$$= \lim_{x \to 0^-} \quad \alpha^c \exp\{-(-\alpha x)^c\}/\exp\{-(-x)^c\} \qquad \text{(using l'Hopital's rule)}$$

$$= \alpha^c$$

This shows that the asymptotic distribution assumed by largest values from a Reverse Weibull distribution is also Reverse Weibull.

A stronger result is, however, available. Let F be the Reverse Weibull parent cumulative distribution function (c.d.f.). Then the c.d.f. of the largest value of a sample of size n drawn from F is

$$\begin{aligned} G(x) &= (F(x))^n \; [= p\,(x_1, x_2, \ldots.\, x_n \leq x)] \\ &= [\exp\{-((a-x)/b)^c)\}]^n \\ &= \exp\{-\,n((a-x)/b)^c\} \\ &= \exp\{-((a-x)/n^{-(1/c)}b)^c\} \end{aligned} \tag{6}$$

which is again exactly Reverse Weibull, with the same location (a) and shape (c) parameters, but with scale parameter now $b^* = bn^{-1/c}$.

In summary, for any distribution satisfying (4), the distribution of maximum values from a sample of size n approaches a Reverse Weibull distribution as n increases. In the special case where the parent distribution is Reverse Weibull, the distribution of maximum values is exactly Reverse Weibull, for any sample size n.

DISCUSSION

It is important to realise the relationships between the parent Reverse Weibull and samples of maxima from it also being Reverse Weibull are coincidental; disregarding degenerate cases, then any distribution that is bounded from above and satisfies the Gnedenko criterion, will produce sample maxima which tend to a Reverse Weibull distribution. For a Reverse Weibull, however, an asymptotic derivation can be replaced by an exact solution, which holds for any sample of size n.

The rationale underlying the principles outlined by Kuru et al. (1991) remains valid. If the maximal distributions are regarded as:

$$G(x) = \exp(-((a^*-x)/b^*)^{c^*} \qquad (7)$$

in the limiting or exact cases, then, given estimates of a^*, b^*, and c^*, percentiles of (7) can be chosen through:

$$X_p = a^*-b^* \, (-\ln(p))^{(1/c^*)} \qquad (8)$$

and equating $\hat{a} = d_{max(p)}$, for chosen p, in (1), then evaluating the resultant goodness-of-fit of stand table data at various ages.

What is less clear is the most efficient way of estimating the parameters in (7). Currently, b^*, c^* are estimated by moments from projection equations of maximum diameter average and standard deviation, and a^* estimated through a projection equation of absolute maximum diameter. Theoretically, far simpler and more direct estimates should be available through directly equating a^* and c^* to estimates obtained through the first stage of Kuru's process (their equations 8(a), 8(b), and 16). Estimation of b^*, however, depends in part on n, the sample size. Study is continuing as to how the latter should be evaluated. The situation is analogous to cluster sampling with unequal sample sizes, Cochran (1981); what is not obvious is how to account for the degree of intra-cluster correlation between trees within sample plots, which has a direct bearing on the derived value of n.

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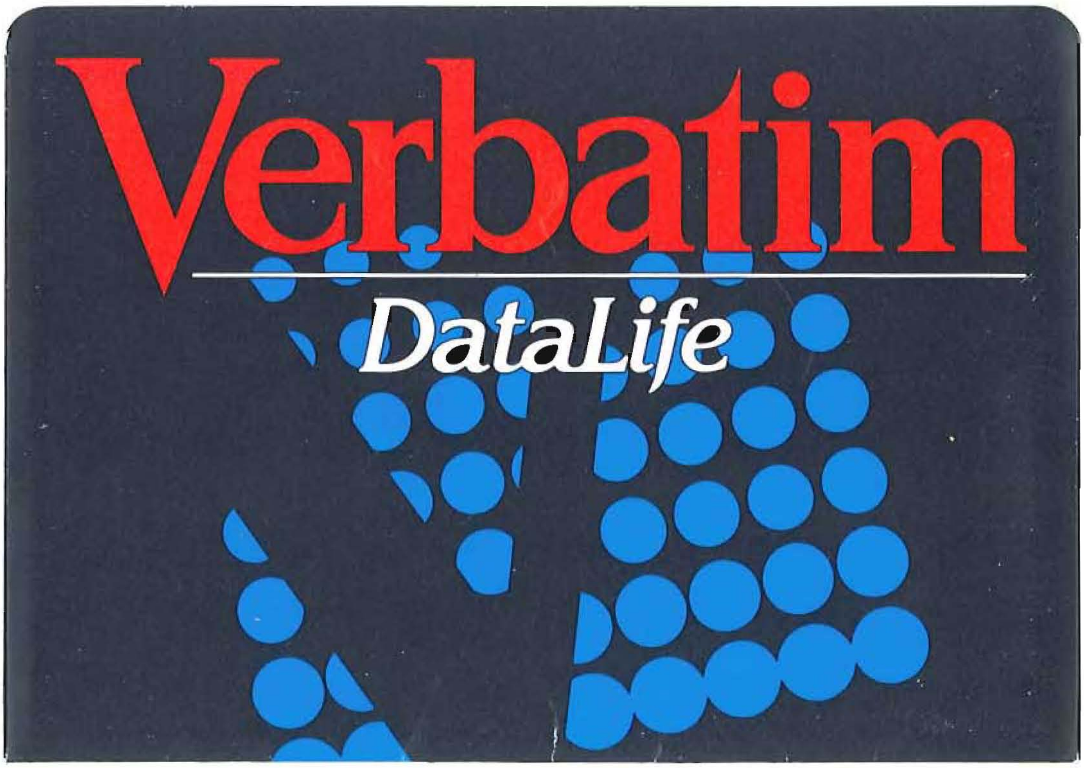
Appendix 4 Programs DFCNIGM1, DFCNIGM2, DFCNIGM3 (floppy disks)



Appendix 5 Coefficients of volume and merchantable volume equations

b_0 = -10.03201;

b_1 = 1.79068;

b_2 = 1.7473;

b_3 = 0.554762;

b_4 = 0.0;

b_5 = 12.38248;

b_6 = -19.0289;

b_7 = 8.43354;

b_8 = 0.870051;

b_9 = 31;

b_{10} = 0.277381;

b_{11} = 0.0;

b_{12} = 3.09562;

b_{13} = -3.80578;

b_{14} = 1.40559;

b_{15} = 0.027189;

Appendix 6 Instructions for running the programmes

DFCNIGM1 is started, at the DOS prompt, by entering DFCNIGM. A short description of the model is then given, followed by a series of questions to which the user must respond. The first such prompt on the screen asks the user if a hardcopy (or printout on the printer) is required: if yes, Y is entered, N if not. The next request is for 2 or 3 numerical starting values, namely site index, age , mean top height at that age: if any of these values is unknown, enter 0 (zero). Given any 2 of these 3 values, the third can be derived and then displayed on the screen; if all these have non-zero values, a reconciliation for consistency is made, but then the program operates on the given mean top height and age entries. The remaining stand starting values are initial stocking (N/ha) and initial basal area/ha for the specified starting age, separated by a comma.

At this point, there are different branches to the program, depending upon the inputs entered. For stands infected with Swiss needle cast, Phaeocryptopus gaeumannii, a Y is entered at the prompt, N if not on either branch. The next question relates to whether or not the stand has been or will be thinned, each category requiring a Y entry whereas for neither, N applies. For this Y subbranch, thinning can be simulated or the stand grown to any specified age within reason either backwards or forwards in time . For the N Sub-branch, the stand may only be grown backwards or forwards without any thinning whatsoever.

Initial output at this point provides a summary of the starting values that have been just entered and then the start of the yield tabulations of mean top height, stocking/ha, basal area/ha, mean dbhob, total stem volume/ha and merchantable volume to a 15 cm top end diameter for input age. Output appears on the screen automatically and simultaneously on the printer if the response was Y to a hardcopy. The stand can then be grown forward or backward year by year after entering an age in years, or thinned at any time through entering a T, or stopped if an S is entered.

Where T is the response, the following data should be supplied by the user: basal area/ha and/or number of stems/ha to remain after thinning. The former is the preferred option. Output then relates to thinnings removed in terms of stocking, basal area, mean dbhob, total stem and merchantable volumes. The simulation of the stand growth can then be conducted backwards or forwards through time.

The same options are available for the thinned, undiseased stands, while, for unthinned stands the backwards and forwards growth and yield simulations are available but not a thinning option. To carry out another simulation, Y can be used when prompted with the question.

DFCNIGM2 is run in the same way as DFCNIGM1 except following differences:

(1) at DOS prompt enter DF rather than DFCNIGM;

(2) a few more inputs are required, namely, basal are/ha, stems/ha before or after thinning or both;

(3) the backward solution is not available in this program.

The way to run DFCNIGM3 is the same as DFCNIGM2 except DFCNIGM3 requires additional inputs, namely standard deviation of the stand and the extreme values: mean, maximum and standard deviation of the largest diameters. They should be calculated in the following way: first extract the largest diameter of each observation. Then sort those largest diameters by age and calculate the mean, maximum and standard deviation for each age. The inputs of extreme values should be those with same initial age as used to start the programme.

Appendix 7 Data disks containing files from which the models were developed.

The data files have been made into archives under the names liu1 and liu2. To use the files, one should type, at DOS prompt, either liu1 or liu2 and press return. The files will be unloaded into the hard disk. They can be used on the PC straightaway or copied onto mainframe.